Matrix models - Practice test 1

10 questions | 45 marks | 70 minutes

Core techniques, network interpretation and two-state forecasts. Network matrices use rows=start and columns=destination; transition matrices use columns=current and rows=next unless stated otherwise.

Equivalent correct methods earn the corresponding marks.

1. Matrices have rows A: $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, and B: $\begin{pmatrix} 1 & 5 \\ -2 & 0 \end{pmatrix}$. [5 marks]

(a) Find $A + B$. [2 marks]

Add corresponding entries.

Answer: Rows $\begin{pmatrix} 3 & 4 \\ 1 & 4 \end{pmatrix}$.

(b) Find $2A - B$. [3 marks]

Double A, then subtract corresponding entries of B.

Answer: Rows $\begin{pmatrix} 3 & -7 \\ 8 & 8 \end{pmatrix}$.

2. A shop sells two products at AUD 4 and AUD 7. Quantities sold on three days are $(5, 2), (3, 4)$, $(6, 1)$. [5 marks]

(a) State the orders of the sales matrix and price vector. [2 marks]

Rows represent days; columns represent products.

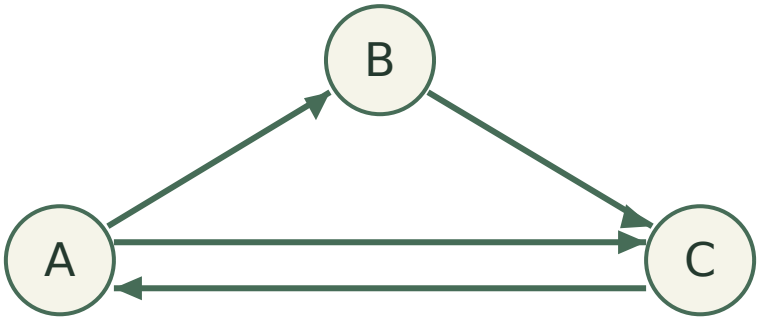
Answer: 3×2 and 2×1.

(b) Use a matrix product to calculate the three daily incomes. [3 marks]

Compute $5(4) + 2(7) = 34$, $3(4) + 4(7) = 40$, $6(4) + 1(7) = 31$.

Answer: Column $\begin{pmatrix} 34 \\ 40 \\ 31 \end{pmatrix}$ dollars.

3. The diagram shows direct links, ordered A,B,C. [4 marks]



Each arrow is one permitted move.

(a) Write the adjacency matrix. [3 marks]

Record every arrow in its starting row and destination column.

Answer: $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

(b) State the number of links entering C. [1 mark]

Sum column C.

Answer: 2.

4. Use adjacency rows $\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$, ordered A,B,C. [5 marks]

(a) Calculate A^2. [3 marks]

Row 1 entries: $0(0)+1(0)+1(1)=1;\ 0(1)+1(0)+1(0)=0;\ 0(1)+1(1)+1(0)=1.$

Row 2 entries: $0(0)+0(0)+1(1)=1;\ 0(1)+0(0)+1(0)=0;\ 0(1)+0(1)+1(0)=0.$

Row 3 entries: $1(0)+0(0)+0(1)=0;\ 1(1)+0(0)+0(0)=1;\ 1(1)+0(1)+0(0)=1.$

So A^2 has rows $\begin{pmatrix}1&0&1\\1&0&0\\0&1&1\end{pmatrix}$.

Answer: $\begin{pmatrix}1&0&1\\1&0&0\\0&1&1\end{pmatrix}$

(b) Interpret entry row A, column C of A^2 and identify the walk. [2 marks]

The entry equals 1 and counts exactly two steps.

Answer: One two-step walk: A→B→C.

5. An unweighted directed network is the chain P→Q→R→S. [4 marks]

(a) Is S reachable from P? Which matrix power first records it? [2 marks]

The shortest route uses three edges.

Answer: Yes; A^3.

(b) Can S reach P? Explain. [2 marks]

No arrow points back along the chain.

Answer: No.

6. A beats B and C; B beats C; C has no wins. Score by row sums of $D + 0.5D^2$. [5 marks]

(a) Calculate the direct-win scores. [2 marks]

Count the outgoing dominance arrows.

Answer: $(2, 1, 0)$.

(b) Calculate the weighted scores and rank the teams. [3 marks]

Only A→B→C adds a two-step score of 0.5.

Answer: $(2.5, 1, 0)$; A, B, C.

7. A two-state transition matrix has rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, in A,B order. [4 marks]

(a) Interpret the entry 0.3. [1 mark]

It is destination row A, starting column B.

Answer: 30% of those currently in B move to A next period.

(b) Predict the next state from $(120, 80)$. [3 marks]

A: $0.8(120) + 0.3(80) = 120$.

B: $0.2(120) + 0.7(80) = 80$.

Answer: $(120, 80)$.

8. A proposed closed transition matrix has rows $\begin{pmatrix} 0.6 & p \\ q & 0.75 \end{pmatrix}$. [4 marks]

(a) Find p, q. [2 marks]

Set each column sum equal to 1.

Answer: $p = 0.25, q = 0.4$.

(b) Is column summation alone sufficient to check a transition matrix? Explain. [2 marks]

Negative entries can still occur in columns totalling 1.

Answer: No; all entries must also be probabilities between 0 and 1.

9. For T with rows $\begin{pmatrix}0.8 & 0.3\\0.2 & 0.7\end{pmatrix}$, let $x_0=(100,0)$. [4 marks]

(a) Find x_3. [3 marks]

The successive states are $(80,20),(70,30),(65,35)$.

Answer: $(65,35)$.

(b) Explain why T^3 alone is not the population forecast. [1 mark]

The population depends on the starting distribution.

Answer: It is a transition matrix; multiply by the initial vector.

10. A bike model uses rows $\begin{pmatrix}0.7 & 0.2\\0.3 & 0.8\end{pmatrix}$. There are 250 bikes. [5 marks]

(a) Find steady counts. [3 marks]

Balance gives $0.3a=0.2b$, hence ratio 2:3.

Use total 250.

Answer: $(100,150)$.

(b) Give two assumptions needed for using this as a long-run forecast. [2 marks]

Also assess whether the model approaches this equilibrium.

Answer: The bike population remains closed and the transition rule stays appropriate each day.