Long-term behaviour and steady states

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. For rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, find steady proportions. [3 marks]

$0.2a = 0.3b$ and $a + b = 1$.

Answer: $(0.6, 0.4)$.

2. Use the same model for 350 members. Find steady counts. [3 marks]

Multiply $(0.6, 0.4)$ by 350.

Answer: $(210, 140)$.

3. For rows $\begin{pmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{pmatrix}$, find steady proportions. [3 marks]

$0.1a = 0.4b$ and $a + b = 1$.

Answer: $(0.8, 0.2)$.

4. For the swap matrix, find steady proportions and explain the behaviour from $(1, 0)$. [3 marks]

Balance gives $a = b$.

Applying the swap repeatedly never approaches that balance from $(1, 0)$.

Answer: Steady $(0.5, 0.5)$; the specified start alternates $(1, 0), (0, 1)$.

5. Find steady proportions for rows $\begin{pmatrix} 0.5 & 0.25 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 0.25 & 0.5 \end{pmatrix}$. [3 marks]

The first and third equations give $a = c = 0.5b$.

Normalise using $2b = 1$.

Answer: $(0.25, 0.5, 0.25)$.

6. A closed model has steady counts $(80, 120)$ out of 200. [4 marks]

(a) State the steady proportions. [1 mark]

Divide by 200.

Answer: $(0.4, 0.6)$.

(b) Predict steady counts if the total doubles and the transition rule is unchanged. [2 marks]

Multiply the proportions by 400.

Answer: $(160, 240)$.

(c) Give one reason observed long-run counts might differ. [1 mark]

The equilibrium prediction is conditional on its assumptions.

Answer: The system may open or transition behaviour may change.