Repeated transitions

6 questions | 21 marks

Equivalent correct methods earn the corresponding marks.

1. Use rows $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$ and initial $(100, 0)$. Find x_2. [3 marks]

First obtain $(80, 20)$.

Multiply once more: $(64 + 6, 16 + 14)$.

Answer: $(70, 30)$.

2. For the same matrix, begin at $(0, 100)$. Find x_1, x_2. [3 marks]

First multiply the initial state.

Then A is $0.8(30) + 0.3(70) = 45$ and B is 55.

Answer: $(30, 70)$, then $(45, 55)$.

3. For that matrix, verify that $(60, 40)$ is unchanged after two periods. [3 marks]

A: $0.8(60) + 0.3(40) = 60$.

B: $0.2(60) + 0.7(40) = 40$; repeat the same calculation.

Answer: Both predictions are $(60, 40)$.

4. A swap matrix has rows $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Start at $(40, 10)$. Find the states after 5 and 6 periods. [3 marks]

Odd powers swap entries; even powers restore them.

Answer: After 5: $(10, 40)$; after 6: $(40, 10)$.

5. Use T: $\begin{pmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{pmatrix}$, and U: $\begin{pmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{pmatrix}$, with $x_0 = (100, 0)$. [6 marks]

(a) Calculate UTx_0. [3 marks]

$Tx_0 = (80, 20)$, then apply U.

Answer: $(50, 50)$.

(b) Calculate TUx_0 and explain the difference. [3 marks]

$Ux_0 = (60, 40)$, which T leaves unchanged.

Answer: $(60, 40)$; it reverses the policies.

6. Name two checks after computing $T^{20}x_0$ for a closed 500-person model. [3 marks]

Also check state order and whether the same rule is justified for 20 periods.

Answer: Nonnegative entries and total 500, apart from small rounding error.