Transition matrices and state vectors

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Use rows $\begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$ to update $(80, 20)$. [3 marks]

A: $0.7(80) + 0.2(20) = 60$.

B: $0.3(80) + 0.8(20) = 40$.

Answer: $(60, 40)$.

2. Rows of a proposed closed transition matrix are $(0.4, 0.3), (0.5, 0.7)$. Is it valid? [3 marks]

It does not account for 10% of those initially in A.

Answer: No; the first column totals 0.9.

3. Find a, b so rows $\begin{pmatrix} 0.75 & a \\ b & 0.6 \end{pmatrix}$ form a column-stochastic matrix. [3 marks]

$0.75 + b = 1$ and $a + 0.6 = 1$.

Answer: $a = 0.4, b = 0.25$.

4. A state vector is $(120, 80)$. What total must a closed transition preserve? [3 marks]

The starting entries sum to 200.

Column sums of 1 preserve this total.

Answer: 200.

5. Each day 90% of North bikes stay North; 40% of South bikes move North. [5 marks]

(a) Write the North,South column-stochastic matrix. [2 marks]

Each column describes destinations from one depot.

Answer: Rows $\begin{pmatrix} 0.9 & 0.4 \\ 0.1 & 0.6 \end{pmatrix}$.

(b) Predict tomorrow from $(20, 30)$. [3 marks]

North: $0.9(20) + 0.4(30) = 30$.

South: $0.1(20) + 0.6(30) = 20$.

Answer: $(30, 20)$.

6. A forecast is $(34.6, 15.4)$ for 50 bikes. Explain its meaning. [3 marks]

An operational integer plan should still total 50.

Do not interpret fractional counts literally.

Answer: It is an expected distribution; actual whole-bike counts vary.