Reachability and dominance/ranking applications

6 questions | 19 marks

Equivalent correct methods earn the corresponding marks.

1. List reachable ordered pairs of different vertices in A→B→C. [3 marks]

The first and last are direct; A→B→C supplies the indirect connection.

Answer: (A, B), (A, C), (B, C).

2. Dominance rows are $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Find direct scores and scores from $D + D^2$. [3 marks]

Row sums of D give direct wins.

Only A has one two-step win, via B to C.

Answer: Direct $(2, 1, 0)$; combined $(3, 1, 0)$.

3. A beats B, B beats C and C beats A. Find direct scores and explain the tie. [3 marks]

Each row contains one direct win.

Answer: Each team scores 1; the results form a cycle.

4. For the same cycle, use row sums of $D + 0.5D^2$. Find all scores. [3 marks]

Each team has one direct win and one two-step dominance walk.

Answer: All are 1.5.

5. Which adjacency powers suffice to test reachability between different vertices in a five-vertex network? [3 marks]

A route with no repeated vertices has at most four edges.

Any longer reachable route can have cycles removed.

Answer: A, A^2, A^3, A^4.

6. Club A gets 40 votes from 100 eligible voters; B gets 30 from 50. [4 marks]

(a) Find each support percentage. [2 marks]

Calculate $\frac{40}{100}$ and $\frac{30}{50}$.

Answer: A: 40%; B: 60%.

(b) Explain why a ranking rule must name its denominator. [2 marks]

Different criteria answer different questions.

Answer: A wins by vote count, but B wins by support rate.