Powers and numbers of connections

6 questions | 18 marks

Equivalent correct methods earn the corresponding marks.

1. For rows A: $\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$, find A^2. [3 marks]

Row 1 entries: $0(0)+1(0)+1(1)=1;\ 0(1)+1(0)+1(0)=0;\ 0(1)+1(1)+1(0)=1.$

Row 2 entries: $0(0)+0(0)+1(1)=1;\ 0(1)+0(0)+1(0)=0;\ 0(1)+0(1)+1(0)=0.$

Row 3 entries: $1(0)+0(0)+0(1)=0;\ 1(1)+0(0)+0(0)=1;\ 1(1)+0(1)+0(0)=1.$

So A^2 has rows $\begin{pmatrix}1&0&1\\1&0&0\\0&1&1\end{pmatrix}$.

Answer: $\begin{pmatrix}1&0&1\\1&0&0\\0&1&1\end{pmatrix}$

2. For that A, find $(A^3)_{AC}$ and interpret it. [3 marks]

Row A of A^2 is $(1,0,1)$.

Dot it with column C of A, $(1,1,0)$, to obtain 1.

The walk is A→C→A→C.

Answer: 1: one three-step walk from A to C.

3. In the undirected path A-B-C, count two-step walks from A to C and from A to A. [3 marks]

They are A→B→C and A→B→A.

Answer: One of each.

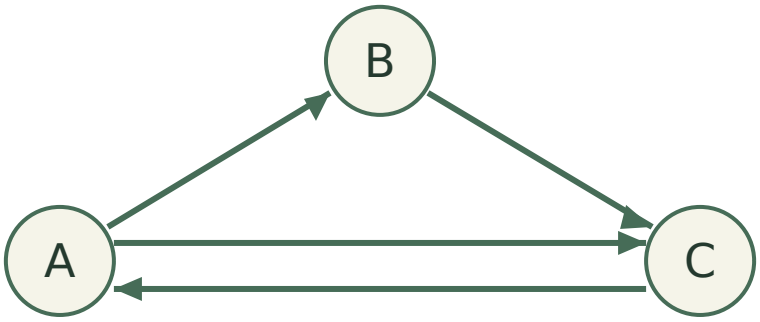
4. Two vertices have one arrow in each direction and no loops. Find the square of their adjacency matrix. [3 marks]

Adjacency rows are $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Two steps return each start to itself.

Answer: The 2×2 identity matrix.

5. For the illustrated network, count A-to-C walks using one or two steps. [3 marks]



Each arrow is one permitted move.

The direct count is 1.

The two-step count is 1. Add the alternatives.

Answer: 2.

6. Why does a positive entry in A^5 not guarantee a journey through five different vertices? [3 marks]

A walk may revisit a vertex or edge.

Answer: The exponent counts edges used, not distinct vertices.