Network/adjacency matrices

6 questions | 20 marks

Equivalent correct methods earn the corresponding marks.

1. Construct the adjacency matrix, ordered A,B,C, for A→B, B→A, B→C and C→C. [3 marks]

Place a 1 for every stated arrow.

C→C belongs on the diagonal.

Answer: Rows $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

2. Rows P,Q,R are $\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Entries count direct bus services. Interpret the 2. [3 marks]

Use row P and column Q.

The weight is a count, not a probability or a duration.

Answer: Two direct services from P to Q.

3. Rows A,B,C are $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Can this represent an undirected network? List its links. [3 marks]

The matrix is symmetric; each link appears in both directions.

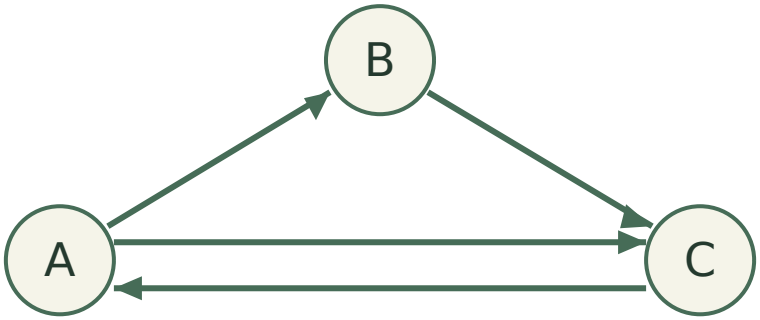
Answer: Yes: A-B and A-C.

4. Explain why zero diagonal entries do not rule out cycles. [3 marks]

A→B→A is a two-step return without either vertex having a loop.

Answer: They rule out one-step loops only. Longer walks can return to the start.

5. Use the diagram and vertex order A,B,C. [5 marks]



Each arrow is one permitted move.

(a) Write its adjacency matrix using row=start. [3 marks]

Read all arrows leaving each vertex.

Answer: Rows $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

(b) How many arrows leave A and arrive at C? [2 marks]

Sum row A and column C respectively.

Answer: Two leave A; two arrive at C.

6. Someone swaps the first two rows of an A,B,C adjacency matrix but not its columns, then calls both orders B,A,C. Explain the error. [3 marks]

The old column A is wrongly being treated as column B.

Answer: Its row and column labels no longer agree. Both must be reordered consistently.