Matrix arithmetic revision

6 questions | 23 marks

Equivalent correct methods earn the corresponding marks.

1. For rows A: $\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, and B: $\begin{pmatrix} 1 & 5 \\ -2 & 0 \end{pmatrix}$, find $2A - B$. [3 marks]

Double A to $\begin{pmatrix} 4 & -2 \\ 6 & 8 \end{pmatrix}$.

Subtract corresponding entries.

Answer: Rows $\begin{pmatrix} 3 & -7 \\ 8 & 8 \end{pmatrix}$.

2. For rows A: $\begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 2 \end{pmatrix}$, and column $b = (4, 1, 2)$, find Ab. [3 marks]

First row: $1(4) + 2(1) + 0(2) = 6$.

Second row: $-1(4) + 3(1) + 2(2) = 3$.

Answer: Column $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$.

3. Let A have rows $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and B have rows $\begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$. [6 marks]

(a) Find AB. [3 marks]

Row 1: $1(2) + 1(1) = 3$ and $1(0) + 1(3) = 3$.

Row 2: $0(2) + 1(1) = 1$ and $0(0) + 1(3) = 3$.

Thus AB has rows $\begin{pmatrix} 3 & 3 \\ 1 & 3 \end{pmatrix}$.

Answer: Rows $\begin{pmatrix} 3 & 3 \\ 1 & 3 \end{pmatrix}$.

(b) Find BA and compare. [3 marks]

Row 1: $2(1) + 0(0) = 2$ and $2(1) + 0(1) = 2$.

Row 2: $1(1) + 3(0) = 1$ and $1(1) + 3(1) = 4$.

Thus BA has rows $\begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$, which differs from AB.

Answer: Rows $\begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix}$; the products differ.

4. A 4×3 matrix multiplies a 3×2 matrix. State the result order. Is the reverse product defined? [3 marks]

Inner dimensions match in the first product.

For the reverse product, 2 and 4 do not match.

Answer: 4×2; the reverse product is not defined.

5. Equal matrices have rows $(x+2, 5), (4, y-1)$ and $(9, 5), (4, 6)$. Find x, y. [3 marks]

Matching entries give $x+2=9$ and $y-1=6$.

Answer: $x=7, y=7$.

6. A bakery sells muffins at AUD 3 and rolls at AUD 2. Branch X sells $(12, 20)$; Y sells $(8, 30)$. [5 marks]

(a) State the sales-matrix and price-column orders. [2 marks]

Rows represent branches and columns products.

Answer: 2×2 and 2×1.

(b) Find and interpret the income column. [3 marks]

Calculate $12(3)+20(2)=76$ and $8(3)+30(2)=84$.

Answer: $(76, 84)$: X earns AUD 76 and Y earns AUD 84.