Linear models and optimisation - Challenge and applications

Name: _____ Date: _____

10 questions | 49 marks | 75 minutes

A final paper requiring careful model criticism, integer reasoning and parameter comparisons.

A scientific calculator may be used.

1. A linear temperature model gives 18 degrees Celsius 2 hours after sunrise and 30 degrees 6 hours after sunrise. [4 marks]

(a) Find a rule $T = mt + b$, where t is hours after sunrise. [3 marks]

(b) Explain why predicting the temperature at $t = 30$ needs caution. [1 mark]

2. For a purchase model, $3x + 2y = 31$ and $x + y = 12$. Counts must be non-negative integers. [5 marks]

(a) Solve the equations. [3 marks]

(b) If the total cost were 30 rather than 31, what would the solution be? [2 marks]

3. A stall sells an item for AUD 17 and incurs AUD 92 fixed cost plus AUD 6 per item. [5 marks]

(a) Find its profit rule. [2 marks]

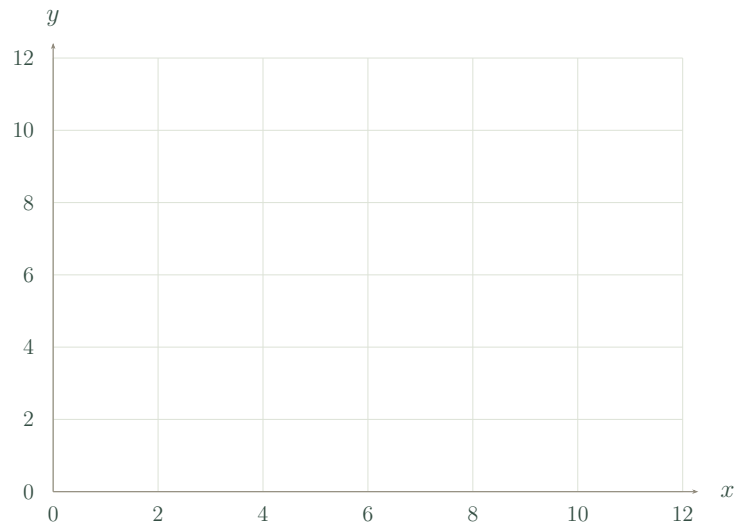
(b) Find the smallest whole-number sales quantity giving at least AUD 100 profit. [3 marks]

4. Solve $5 - 2(3x - 1) > 19 - x$. [4 marks]

(a) Give the solution set. [3 marks]

(b) Find the greatest integer solution. [1 mark]

5. A feasible region satisfies $x \geq 2$, $y \geq 1$, $x + y \leq 10$ and $2x + y \leq 14$. [7 marks]



(a) List all its vertices. [4 marks]

(b) Maximise $P = 7x + 4y$. [3 marks]

6. For the region in Question 5, decisions must instead be whole numbers. [3 marks]

(a) Does the continuous optimum already satisfy integrality? [1 mark]

(b) Explain why it is also an integer optimum without enumerating every integer pair. [2 marks]

7. Minimise $C = 5x + 4y$ with $x, y \geq 0$, $x + y \geq 9$ and $2x + y \geq 13$. [6 marks]

(a) Find the lower-bound vertices. [3 marks]

(b) Find the minimum and interpret unused excess in each minimum requirement. [3 marks]

8. A bounded integer search gives these best feasible y-values for $x = 0, 1, 2, 3, 4$:5,4,4,2,0. Let $P = 4x + 3y$. [5 marks]

(a) Find the maximum over the listed candidates. [3 marks]

(b) What fact must be established before this table proves the global integer optimum? [2 marks]

9. A feasible region is defined only by $x, y \geq 0$. A report gives a finite maximum for $P = 2x + y$. [3 marks]

(a) Explain why the report cannot be correct for this stated region. [2 marks]

(b) State one kind of real information that could justify a finite model. [1 mark]

10. A feasible polygon has vertices $(0,0), (8,0), (4,6), (0,8)$. Profit is $P = px + 4y$, where p is positive. [7 marks]

(a) Find the p-values for which $(4,6)$ is an optimum. [4 marks]

(b) When is $(4,6)$ the unique optimum, and what happens at the endpoints? [3 marks]