Linear models and optimisation - Practice test 1

10 questions | 48 marks | 70 minutes

A balanced paper building from equations and comparisons to constrained decisions.

Equivalent correct methods earn the corresponding marks.

1. A display company charges AUD 65 for setup and AUD 18 per stand. [3 marks]

(a) Define variables and write a linear cost rule. [2 marks]

Let C be dollars and n the number of stands.

65 is fixed; each stand adds 18.

Answer: $C = 65 + 18n$ for whole-number $n \geq 0$.

(b) Find the cost of 9 stands. [1 mark]

$65 + 18(9) = 227$.

Answer: AUD 227.

2. Solve the simultaneous equations $2x + 3y = 24$ and $x + y = 9$. [4 marks]

(a) Find x and y. [3 marks]

Substitute $x = 9 - y$: $18 - 2y + 3y = 24$.

Then $y = 6$ and $x = 3$.

Answer: $(3, 6)$.

(b) Verify both equations. [1 mark]

$2(3) + 3(6) = 24$ and $3 + 6 = 9$.

Answer: Both hold.

3. Plan A costs $A = 48 + 2n$ and plan B costs $B = 18 + 5n$, in dollars. [4 marks]

(a) Find their equal-cost point. [2 marks]

$48 + 2n = 18 + 5n$ gives $n = 10$; substitute.

Answer: 10 uses; AUD 68.

(b) Which plan is cheaper for 15 uses, and by how much? [2 marks]

$A = 78$ and $B = 93$.

Answer: A by AUD 15.

4. A maker can spend at most 42 hours. A small unit uses 2 hours and a large unit 5 hours. [3 marks]

(a) Write the time restriction for x small and y large units. [2 marks]

Add the hours used by each product.

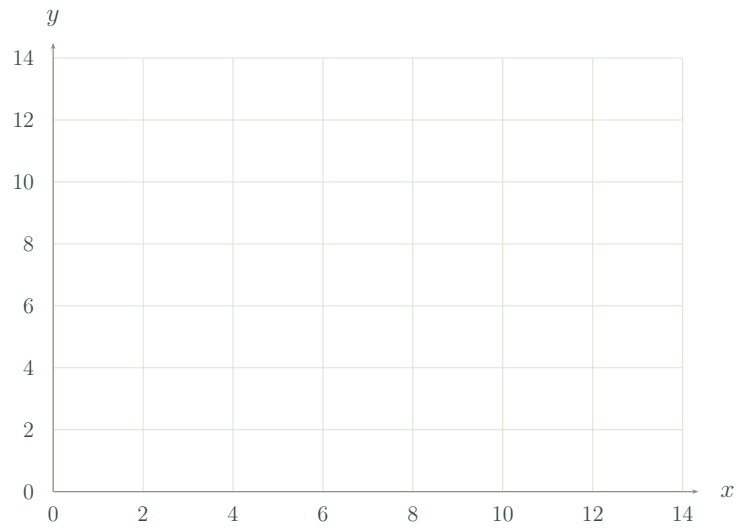
Answer: $2x + 5y \le 42$, with non-negative integer x, y.

(b) Does $(6, 6)$ meet this restriction? [1 mark]

$2(6) + 5(6) = 42$.

Answer: Yes, exactly.

5. Use $x, y \geq 0$, $x + y \leq 12$ and $x \leq 8$. [4 marks]



(a) Sketch the feasible region and list all its vertices. [3 marks]

Draw $x = 8$ and $x + y = 12$; retain the first-quadrant overlap.

Answer: $(0, 0), (8, 0), (8, 4), (0, 12)$.

(b) Explain why $(10, 1)$ is excluded even though its sum is below 12. [1 mark]

Every constraint must hold.

Answer: It violates x at most 8.

6. A shop makes AUD 14 profit on A and AUD 9 on B. The feasible vertices are $(0, 0), (6, 0), (4, 5), (0, 7)$. [4 marks]

(a) Write and evaluate the objective at the vertices. [3 marks]

Use profit coefficients, not quantities alone.

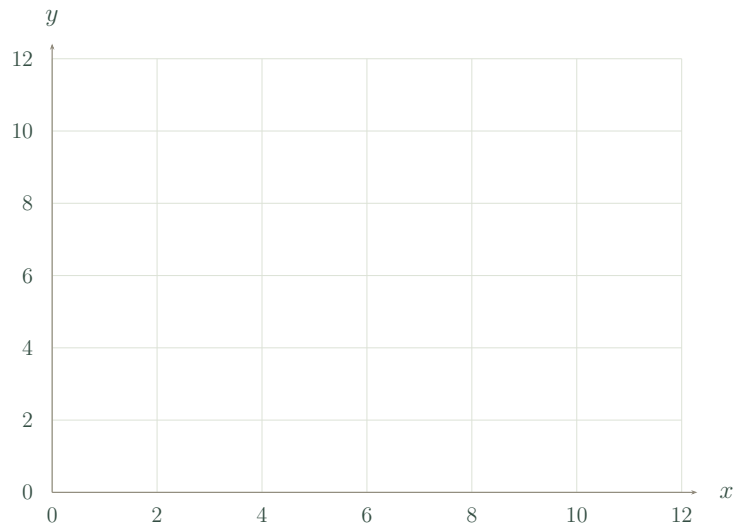
Answer: $P = 14x + 9y$; values 0,84,101,63.

(b) State the best decision and profit. [1 mark]

Choose the largest objective value.

Answer: 4 of A and 5 of B; AUD 101.

7. A workshop makes x small boxes and y large boxes. Each uses one sheet; a small box needs 2 assembly units and a large one 1. There are 10 sheets and 14 assembly units. Unit profits are AUD 5 and AUD 3. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x+y\le 10$, $2x+y\le 14$, $x,y\ge 0$; Maximise $P=5x+3y$. Counts x and y must be integers.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: $(0,0)$, $(0,10)$, $(4,6)$, $(7,0)$.

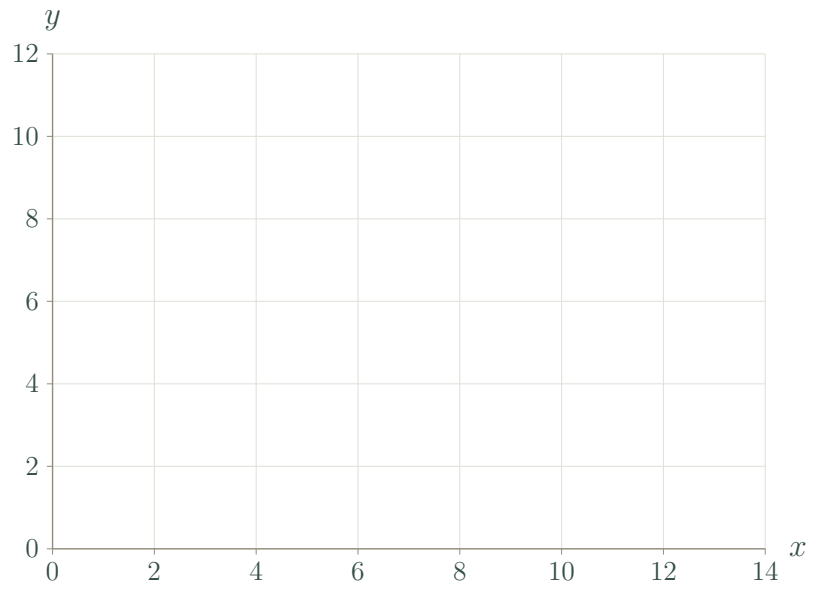
$(0,0)$ gives 0; $(0,10)$ gives 30; $(4,6)$ gives 38; $(7,0)$ gives 35

The largest vertex value is 38.

The continuous optimum has whole-number coordinates, so it also gives the optimum for the item counts.

Answer: $(4,6)$; objective 38.

8. A blend uses x litres of A and y litres of B. It must total at least 8 litres and supply at least 12 nutrient units; A has 1 unit/litre and B has 2. Costs are AUD 3 and AUD 4 per litre. [8 marks]



(a) State the model, including variable restrictions. [3 marks]

Define x and y as described.

Translate each separate resource or minimum requirement.

Answer: $x+y\geq 8$, $x+2y\geq 12$, $x,y\geq 0$; Minimise $C=3x+4y$.

(b) Find and justify the optimal decision. Give the objective value. [5 marks]

Feasible vertices: $(0,8)$, $(4,4)$, $(12,0)$.

$(0,8)$ gives 32; $(4,4)$ gives 28; $(12,0)$ gives 36

The smallest vertex value is 28.

Along each unbounded horizontal or vertical boundary ray the positive cost increases. The minimum is found among the lower-bound corners.

Answer: $(4,4)$; objective 28.

9. For integer x, y, maximise $P = 3x + 4y$ subject to $2x + y \leq 7$, $x + 2y \leq 7$ and non-negativity. [6 marks]

(a) Show why rounding the continuous corner up to $(3, 3)$ fails. [2 marks]

Each left side is 9, greater than 7.

Answer: Both resource bounds fail.

(b) Find the integer optimum using a table. [4 marks]

For $x = 0, 1, 2, 3$, largest feasible $y = 3, 3, 2, 1$.

Objective values are 12,15,14,13.

Answer: $(1, 3)$ with $P = 15$.

10. A linear model recommends producing 60 gift packs, assuming all packs sell and each uses the same labour time. [4 marks]

(a) State two assumptions that should be checked in practice. [2 marks]

The sales and labour assumptions directly affect profit and feasibility.

Answer: Demand for at least 60 packs; constant labour use per pack.

(b) Orders cap gift-pack demand at 45. Explain the required model change. [2 marks]

The previous recommendation may no longer be feasible.

Answer: Add the supported bound of 45 packs and re-optimise.