Matrices and networks - Applications and reasoning test

Name: _____ Date: _____

10 questions | 60 marks | 75 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

A scientific calculator may be used.

1. A stock matrix is $S=\begin{pmatrix} x & 3 \\ 2 & y \end{pmatrix}$. Its row totals are 10 and 8 items. [6 marks]

(a) Find x and y, explaining which totals you used. [3 marks]

(b) Unit values for its two columns are 4 and 5 dollars. Calculate each row’s stock value. [3 marks]

2. Two orders of two product types cost 31 and 37 dollars. The quantity rows are $(2, 3)$ and $(4, 1)$. Let the unit prices be p and q. [6 marks]

(a) Form the matrix equation and solve for the prices. [3 marks]

(b) Check the prices in both orders and explain why the result should be interpreted as a model assumption. [3 marks]

3. Rows of $Q = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 4 & 2 \end{pmatrix}$ describe two bundles. Product prices at suppliers 1 and 2 are columns of $P = \begin{pmatrix} 2 & 5 \\ 3 & 4 \\ 1 & 2 \end{pmatrix}$ dollars. [6 marks]

(a) Calculate QP and interpret all four entries. [3 marks]

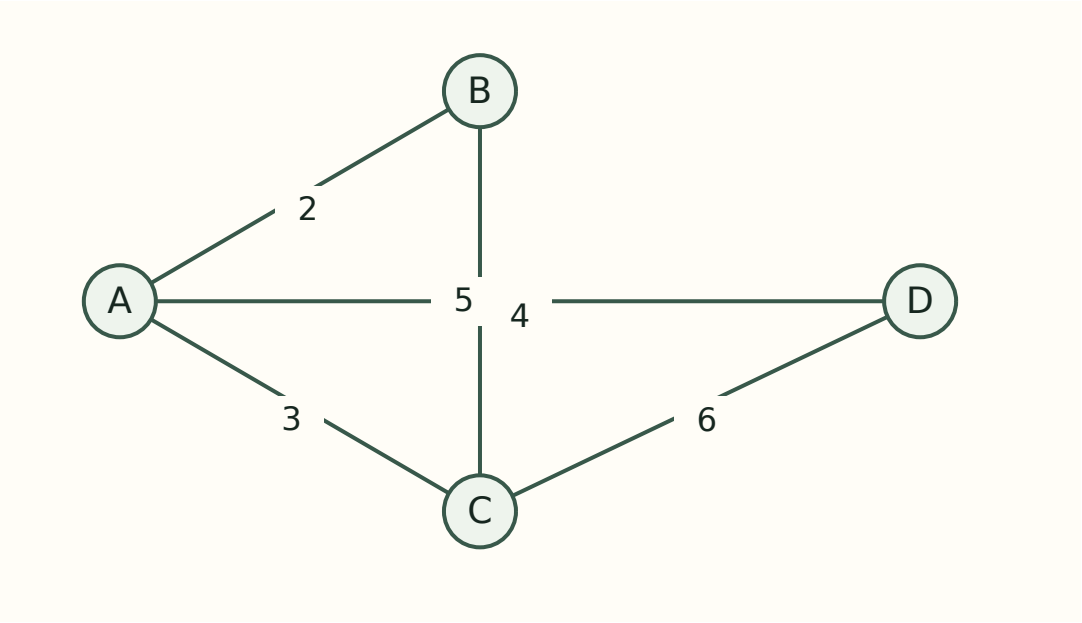
(b) Supplier 1 charges 5 dollars delivery per bundle, supplier 2 none. Recommend a supplier for each bundle. [3 marks]

4. A theatre’s ticket-quantity rows are matinee/evening and columns adult/student. $Q = \begin{pmatrix} 12 & 8 \\ 20 & 15 \end{pmatrix}$. Adult tickets cost 18 dollars; student tickets 12 dollars. [6 marks]

(a) Use a matrix product to find revenue for each performance. [3 marks]

(b) A sponsor pays 2 dollars for each student ticket. Find total revenue including this contribution, and state an assumption. [3 marks]

5. The undirected weights are cable-installation costs. [6 marks]



(a) Find a minimum spanning tree and identify its bridges. [3 marks]

(b) A backup B-D edge is added. Which selected original edge remains a bridge, and why? [3 marks]

6. The directed transport links are listed. Time is minutes; capacity is passengers/min. [6 marks]

All available directed transport links

Link	Time	Capacity
S→A	2	4
A→T	3	4
S→B	4	7
B→T	4	7
A→B	1	2

(a) Find a fastest S-to-T path and its route bottleneck. [3 marks]

(b) Find maximum total throughput using both main branches. Explain why it differs from the fastest route’s capacity. [3 marks]

7. Rows/columns A, B, C in $N=\begin{pmatrix}0 & 2 & 1\\ 0 & 0 & 3\\ 0 & 0 & 0\end{pmatrix}$ count distinct scheduled services between stops.

[6 marks]

(a) Count exactly two-service journeys A-to-C and explain the calculation. [3 marks]

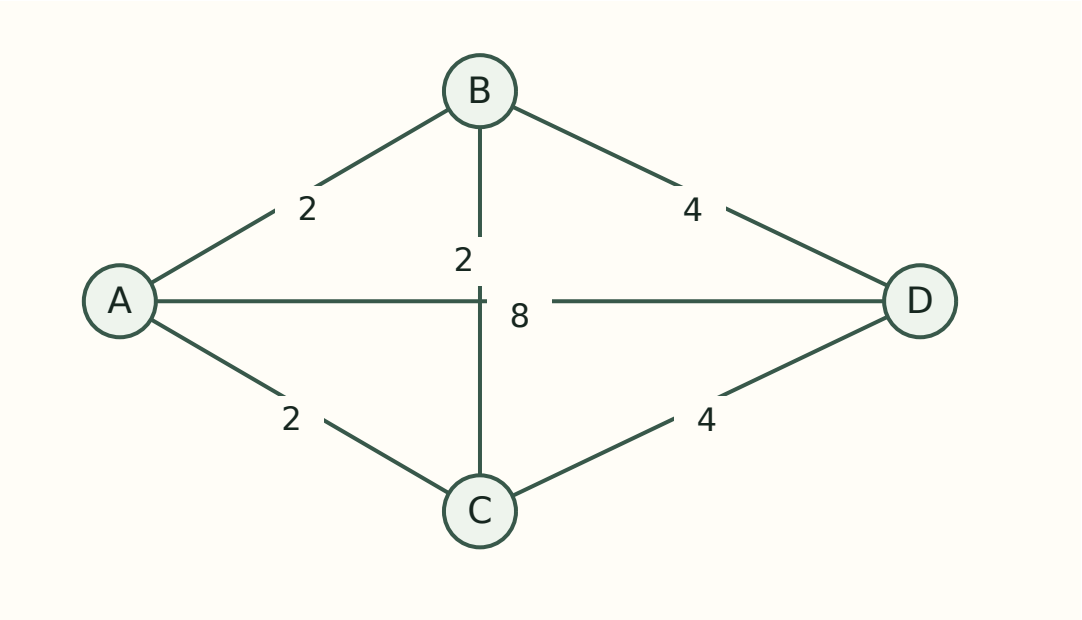
(b) Count all A-to-C service choices with no repeated stop, allowing one or two services. Why would the same matrix calculation not count journeys if its entries were travel times? [3 marks]

8. Directed edges are S→A 3, S→B 4, A→C 5, B→C 1, A→T 6, C→T 4 and B→T 7. [6 marks]

(a) Find a longest S-to-T path in this acyclic graph and justify your answer. [3 marks]

(b) A positive-weight edge C→A is added. Explain why a longest walk, with repetition permitted, now has no finite maximum. [3 marks]

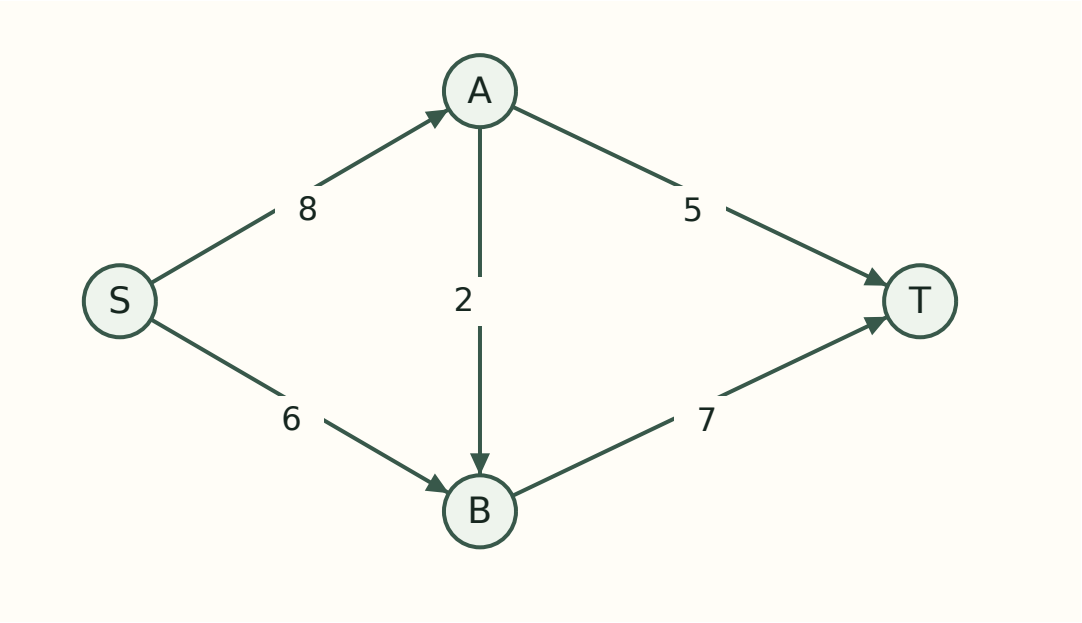
9. Use the undirected weighted network shown. [6 marks]



(a) Find one minimum spanning tree and show why its total cannot be less. [3 marks]

(b) How many different minimum trees are possible? Explain systematically. [3 marks]

10. Pipe capacities are units/hour. You may increase exactly one capacity by 2. [6 marks]



(a) Find and certify the original maximum flow. [3 marks]

(b) Choose between widening S→A or A→T by 2. Prove which gives the larger new maximum. [3 marks]