Matrices and networks - Practice test 2

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

Equivalent correct methods earn the corresponding marks.

1. $A=\begin{pmatrix}2 & x\\ 5 & 1\end{pmatrix}$ and $B=\begin{pmatrix}7 & 9\\ 3 & 4\end{pmatrix}$. [4 marks]

(a) Find x if the top-right entry of $A+B$ is 15. [2 marks]

$x+9=15.$

Subtract 9.

Answer: $x=6.$

(b) Find the remaining three entries of $A+B$. [2 marks]

Add the corresponding known entries.

$2+7=9,\ 5+3=8,\ 1+4=5.$

Answer: Top-left 9; bottom-left 8; bottom-right 5.

2. Opening stock $S=\begin{pmatrix}20 & 15\\12 & 18\end{pmatrix}$, deliveries $D=\begin{pmatrix}4 & 8\\6 & 3\end{pmatrix}$ and sales $R=\begin{pmatrix}9 & 7\\5 & 11\end{pmatrix}$. [4 marks]

(a) Find closing stock $S+D-R$. [2 marks]

Add delivery quantities then subtract sales.

Answer: $\begin{pmatrix}15 & 16\\13 & 10\end{pmatrix}$.

(b) Use a total-stock check to verify the result. [2 marks]

Opening total 65, deliveries 21, sales 32.

$65+21-32=54$; result entries also sum 54.

Answer: Closing total 54 items.

3. Let $A=\begin{pmatrix}1 & 2 & 3\\0 & 4 & 1\end{pmatrix}$ and $B=\begin{pmatrix}2\\1\\5\end{pmatrix}$. [4 marks]

(a) State the product order and calculate AB. [2 marks]

$1(2)+2(1)+3(5)=19$.

$0(2)+4(1)+1(5)=9$.

Answer: 2×1; $\begin{pmatrix}19\\9\end{pmatrix}$.

(b) Does BA exist? Explain. [2 marks]

The orders would be $(3\times1)(2\times3)$.

The inside dimensions 1 and 2 differ.

Answer: No.

4. Rows in $Q=\begin{pmatrix}3 & 4\\5 & 2\end{pmatrix}$ are orders; columns are coffee and tea boxes. Prices are 12 and 8 dollars per box. [4 marks]

(a) Calculate the two costs using a price column. [2 marks]

$3(12)+4(8)=68.$

$5(12)+2(8)=76.$

Answer: $P=\begin{pmatrix}12\\8\end{pmatrix}$; costs 68 and 76 dollars.

(b) Tea rises to 10 dollars per box. Find the increase in the combined cost. [2 marks]

There are $4+2=6$ tea boxes.

Each rises by 2 dollars, giving $6(2)=12$.

Answer: 12 dollars.

5. A simple undirected graph has vertices A, B, C, D, E, F and edges AB, BC, CA, DE. [4 marks]

(a) State the components, including isolated vertices. [2 marks]

A, B, C form a triangle.

D, E are connected; F has no edge.

Answer: {A, B, C}, {D, E}, {F}: three components.

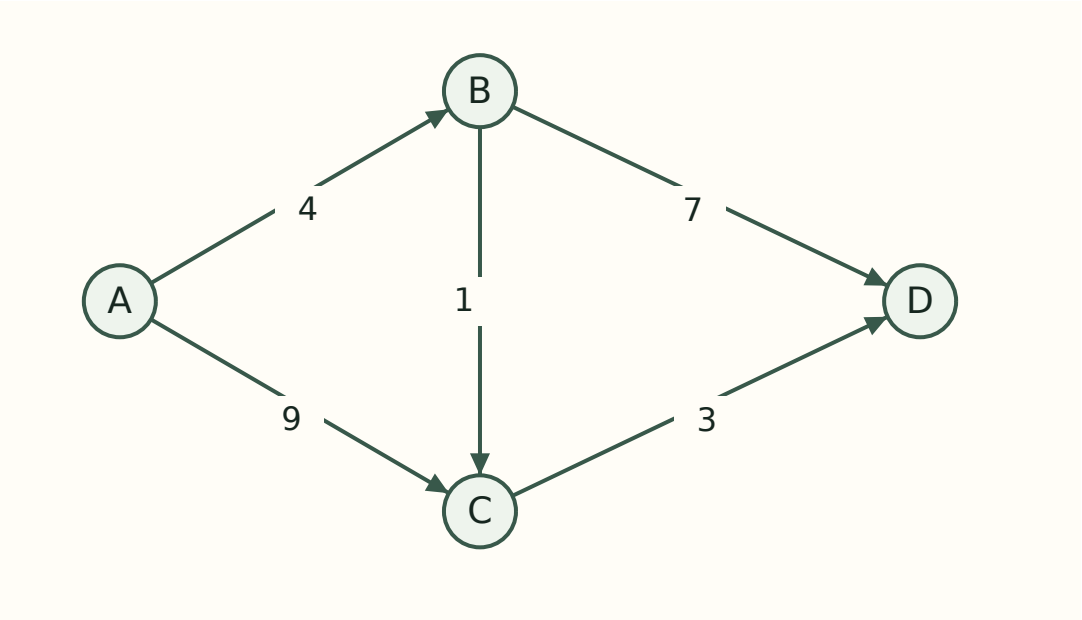
(b) What is the minimum number of new edges needed to connect the whole graph? Give an example. [2 marks]

Three components require at least two joins.

These joins connect all components.

Answer: Two; for example C-D and E-F.

6. The arrows show permitted bus connections; weights are minutes. [4 marks]



(a) Find the duration A→B→C→D and compare with A→C→D. [2 marks]

$4 + 1 + 3 = 8.$

$9 + 3 = 12.$

Answer: 8 min versus 12 min; first saves 4 min.

(b) If B→C is closed, compare the two remaining A-to-D paths. [2 marks]

Remove only the closed arrow.

Choose the 11-minute route under the fixed-time model.

Answer: A→B→D takes 11 min; A→C→D takes 12 min.

7. Edges are A→B, A→C, B→A, B→D and C→D. No other edges exist. [4 marks]

(a) List all A-to-D paths with no repeated vertices. [2 marks]

The first step is B or C.

Returning from B to A is excluded by the no-repeat rule.

Answer: A→B→D and A→C→D.

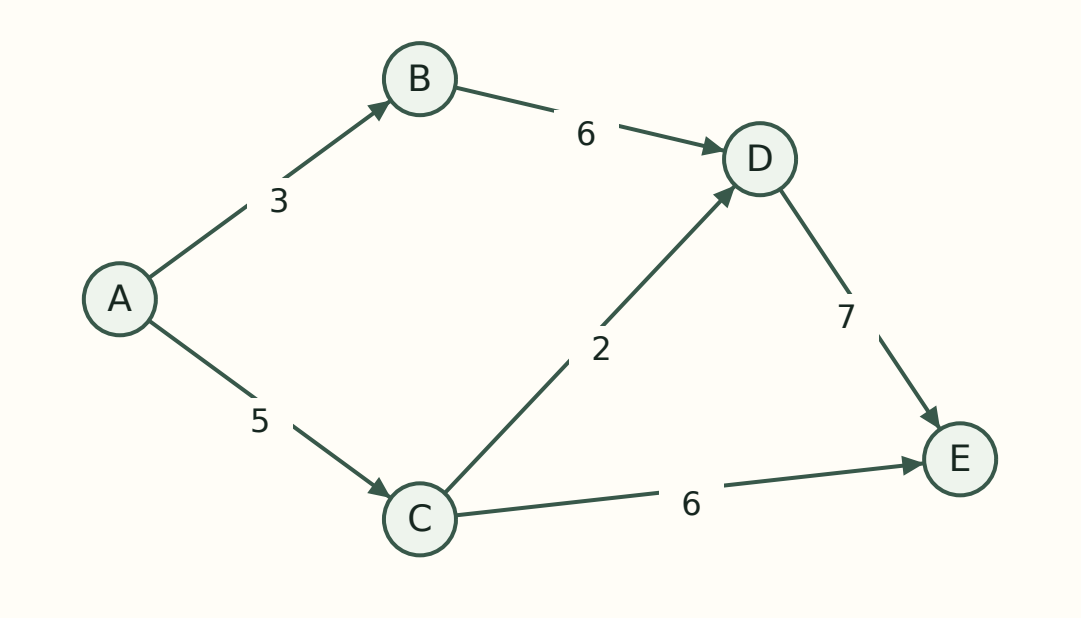
(b) List an A-to-D four-edge walk that is not one of those paths. [2 marks]

Each required arrow exists.

A and B repeat, so it is a walk rather than a simple path.

Answer: A→B→A→B→D.

8. This directed network is acyclic. [4 marks]



(a) Find a longest A-to-E path. [2 marks]

Possible totals are 16, 14 and 11.

Choose the greatest.

Answer: A→B→D→E;16.

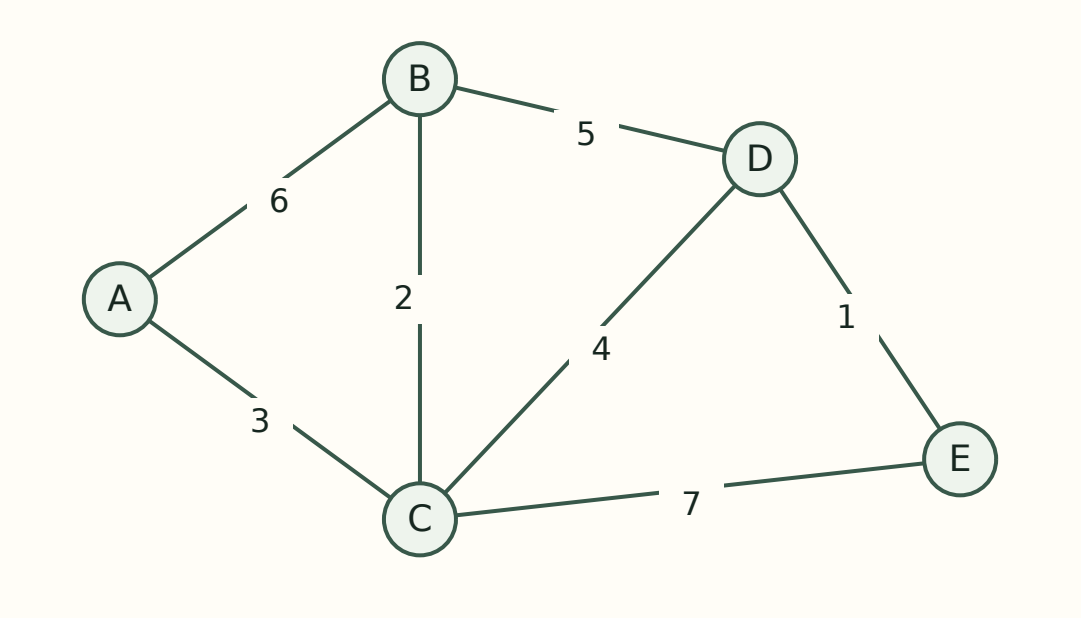
(b) Why would allowing an extra E→A edge of positive weight make the unrestricted longest-walk question different? [2 marks]

A→B→D→E→A is then a cycle.

Every repeat adds positive weight; there is no finite maximum walk.

Answer: A positive directed cycle could be repeated indefinitely.

9. Connection weights are installation costs in thousands of dollars. [4 marks]



(a) Apply Prim from A and report a minimum tree. [2 marks]

Each edge joins the existing set to a new vertex.

The total is $3+2+4+1=10$.

Answer: A-C 3, C-B 2, C-D 4, D-E 1; total 10.

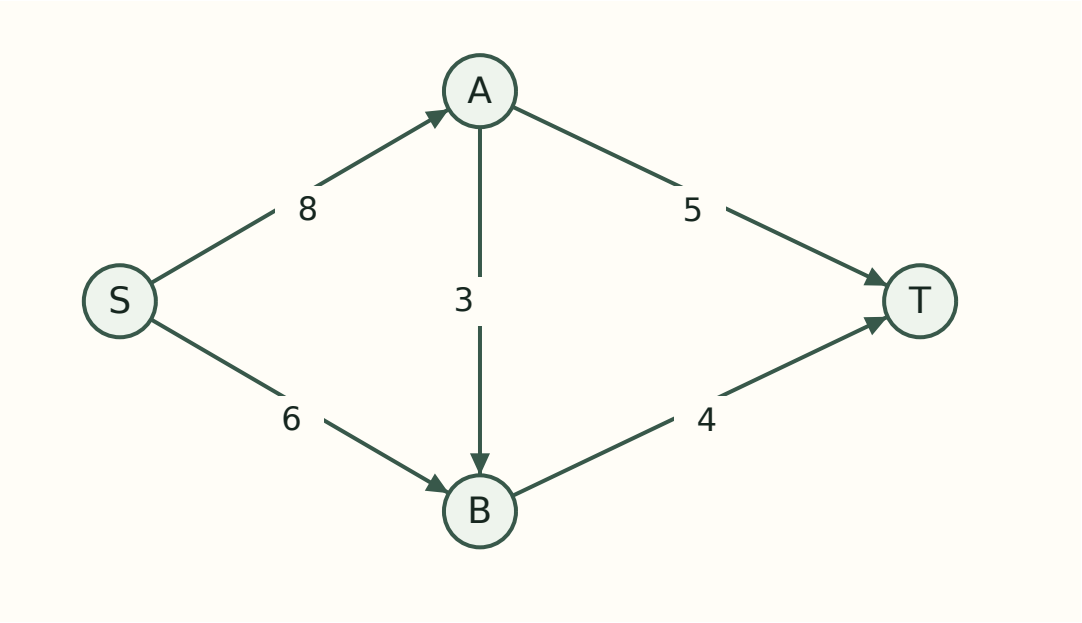
(b) After reaching {A, B, C}, explain why D-E 1 cannot be chosen before C-D 4. [2 marks]

Prim must grow the connected existing set.

It can choose D-E after D or E joins.

Answer: Neither endpoint of D-E is in the current tree.

10. Directed pipe capacities are L/s. [4 marks]



(a) Show that maximum flow is 9 L/s. [2 marks]

This flow obeys capacities and conservation.

The matching sink cut proves the maximum.

Answer: Send SA 5→AT 5 and SB 4→BT 4; sink cut $5+4=9$.

(b) Only S→A is widened from 8 to 12. Does the maximum increase? Explain. [2 marks]

The unchanged sink cut still has capacity 9.

The previous feasible 9 L/s flow still exists.

Answer: No; it remains 9 L/s.