Matrices and networks - Practice test 1

10 questions | 40 marks | 65 minutes

An original mixed chapter paper. Show calculations, state units and justify network decisions using the supplied information.

Equivalent correct methods earn the corresponding marks.

1. A delivery matrix is $D=\begin{pmatrix}12 & 5 & 8\\ 7 & 10 & 3\end{pmatrix}$. Rows are Shop North/South; columns are chairs, desks and lamps. [4 marks]

(a) State its order and total number of entries. [2 marks]

Count 2 rows and 3 columns.

$2(3)=6.$

Answer: 2×3, six entries.

(b) Interpret d_{23} and find the total number of lamps delivered. [2 marks]

The third column records lamps.

$8+3=11.$

Answer: $d_{23}=3$ means 3 lamps for South; total lamps 11.

2. Let $A=\begin{pmatrix}4 & -2\\ 1 & 5\end{pmatrix}$ and $B=\begin{pmatrix}3 & 6\\ -2 & 1\end{pmatrix}$. [4 marks]

(a) Calculate $A+B$. [2 marks]

Add corresponding entries.

Answer: $\begin{pmatrix}7 & 4\\ -1 & 6\end{pmatrix}$.

(b) Calculate $2A-B$. [2 marks]

$2A=\begin{pmatrix}8 & -4\\ 2 & 10\end{pmatrix}$.

Subtract B at matching positions.

Answer: $\begin{pmatrix}5 & -10\\ 4 & 9\end{pmatrix}$.

3. Let $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$. [4 marks]

(a) Calculate AB. [2 marks]

Use row-column products.

First row gives $2(1) + 1(2) = 4$ and $2(4) + 1(0) = 8$.

Answer: $\begin{pmatrix} 4 & 8 \\ 6 & 0 \end{pmatrix}$.

(b) Calculate BA and compare it with AB. [2 marks]

BA uses a different row-column order.

Its first row is $(2, 13)$ and second $(4, 2)$.

Answer: $BA = \begin{pmatrix} 2 & 13 \\ 4 & 2 \end{pmatrix}$; unequal.

4. Two cafes order muffins and juice in quantities $Q = \begin{pmatrix} 8 & 5 \\ 6 & 9 \end{pmatrix}$. Unit prices are $P = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ dollars. [4 marks]

(a) Calculate QP and interpret it. [2 marks]

$8(3) + 5(4) = 44$.

$6(3) + 9(4) = 54$.

Answer: $\begin{pmatrix} 44 \\ 54 \end{pmatrix}$ dollars: the two order costs.

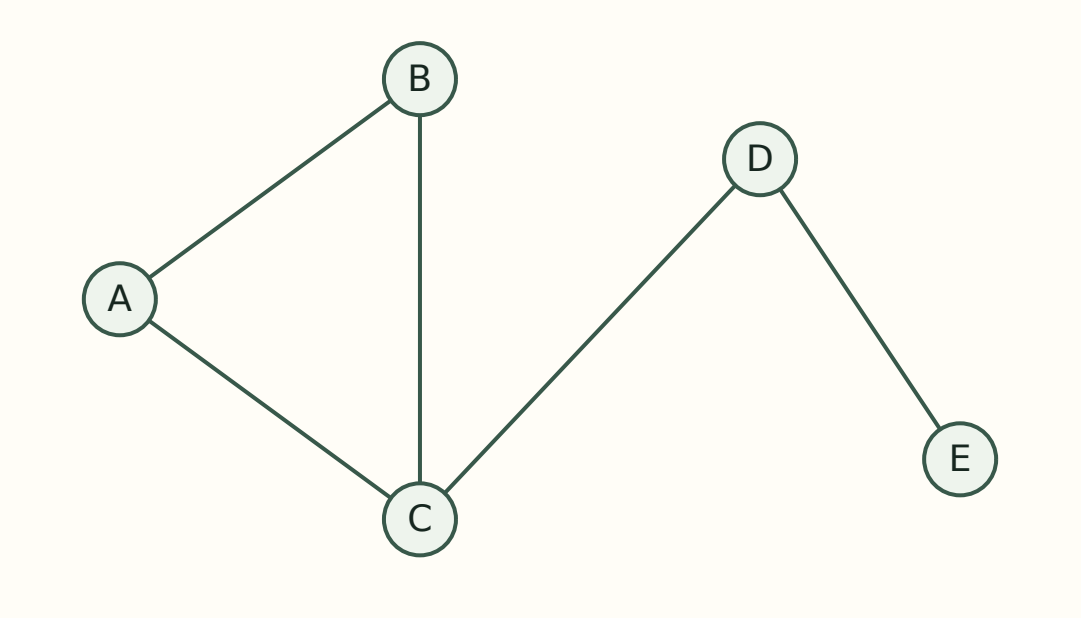
(b) One delivery fee of 6 dollars applies to each cafe. Find the combined payment. [2 marks]

Product costs sum to $44 + 54 = 98$.

Two fees add $2(6) = 12$; total 110.

Answer: 110 dollars.

5. Use the undirected network shown. [4 marks]



(a) Find all vertex degrees and check them against the edge count. [2 marks]

Count the incident edges.

Each edge contributes two endpoints.

Answer: A:2, B:2, C:3, D:2, E:1; degree sum $10 = 2 \times 5$.

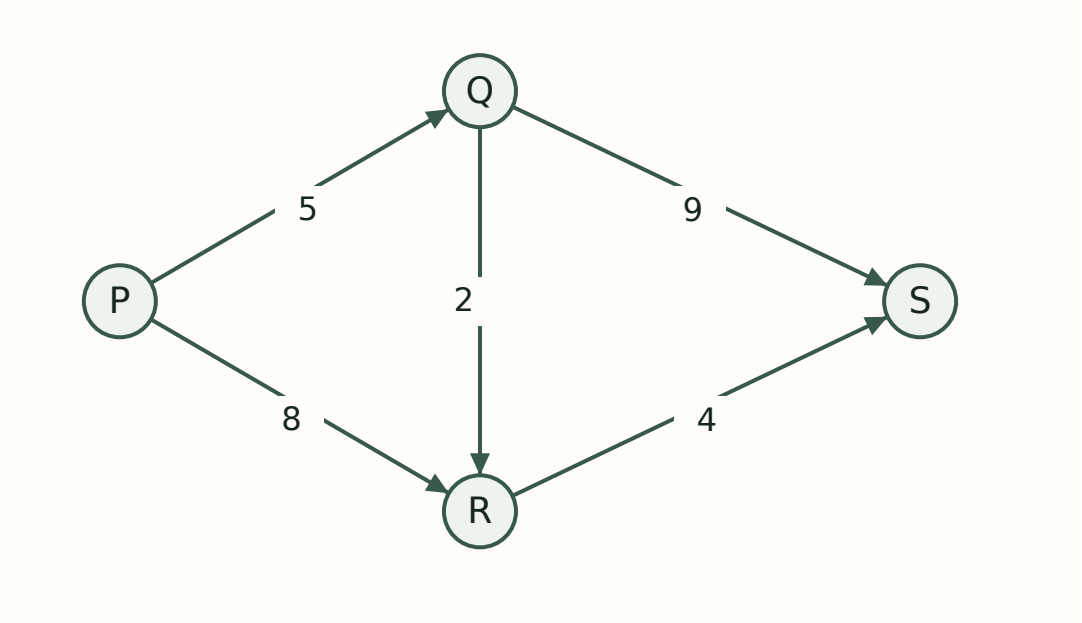
(b) Identify the bridges and explain why they matter. [2 marks]

Removing either disconnects the graph.

No alternative route crosses either separation.

Answer: C-D and D-E.

6. Edge weights in the directed network are minutes. [4 marks]



(a) Find the duration of P→Q→R→S. [2 marks]

All arrows are present.

$5 + 2 + 4 = 11$.

Answer: 11 minutes.

(b) Can S→R→Q→P be used for the return journey? Explain. [2 marks]

The reverse arrows are not supplied.

A forward edge does not imply a reverse edge.

Answer: No.

7. An adjacency matrix has rows/columns A, B, C: $N=\begin{pmatrix}0&1&1\\0&0&1\\1&0&0\end{pmatrix}$. Rows are starts. [4 marks]

(a) Find $(N^2)_{AC}$ and list the associated walk. [2 marks]

Row A times column C gives $0(1)+1(1)+1(0)=1$.

Answer: 1; A→B→C.

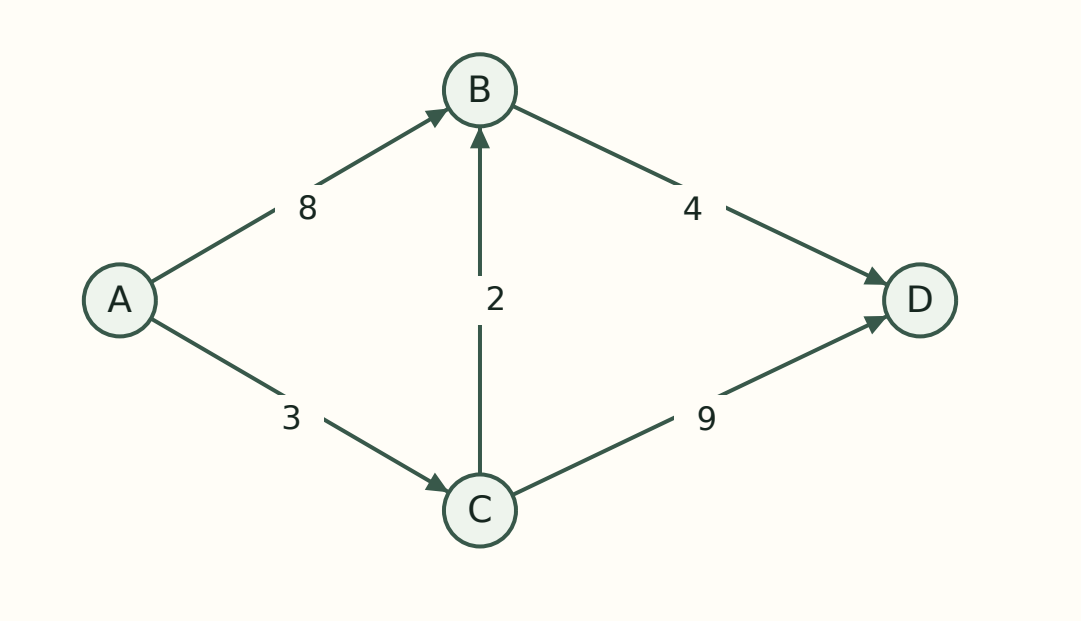
(b) Find $(N^2)_{AA}$ and explain why it does not prove there is a direct loop at A. [2 marks]

The power counts two-edge walks.

A direct loop would be recorded in N_{AA}, which is zero.

Answer: 1; it counts A→C→A.

8. Use the directed weighted graph. [4 marks]



(a) Find a shortest A-to-D path and its weight. [2 marks]

The possible path totals are $8+4=12$, $3+9=12$ and $3+2+4=9$.

Answer: A→C→B→D; 9.

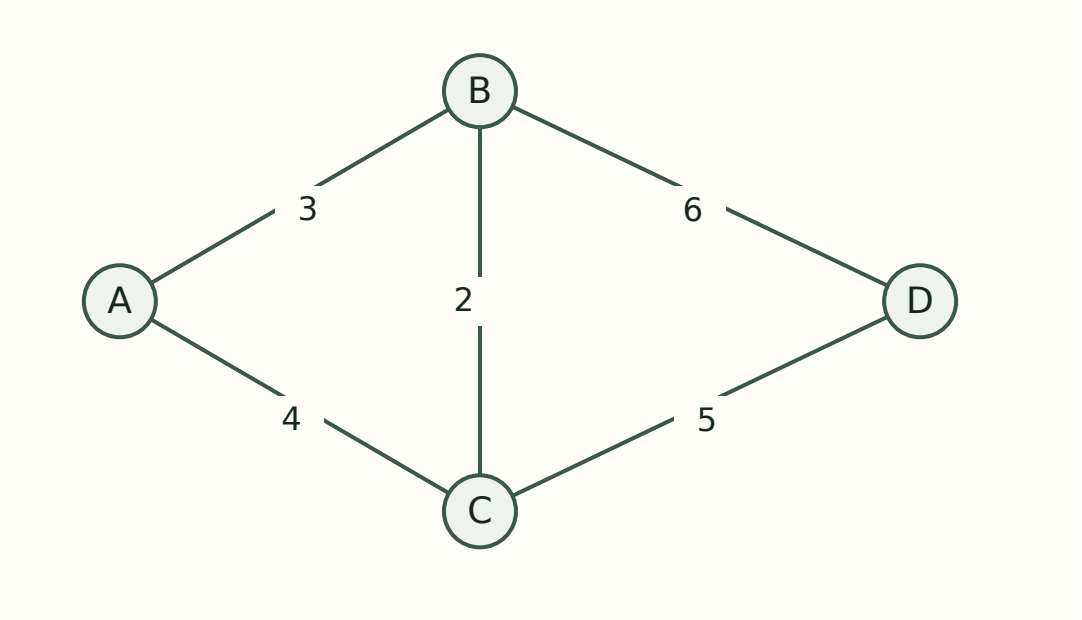
(b) An accident closes C→B. Find a shortest remaining path. [2 marks]

Remove the closed edge.

Both remaining routes have weight 12.

Answer: A→B→D or A→C→D; weight 12.

9. The undirected weights are cable costs in hundreds of dollars. [4 marks]



(a) Use Prim’s algorithm from A to find a minimum spanning tree. [2 marks]

Choose A-B 3, then B-C 2, then C-D 5.

The four vertices are connected by three edges.

Answer: A-B, B-C, C-D; weight 10.

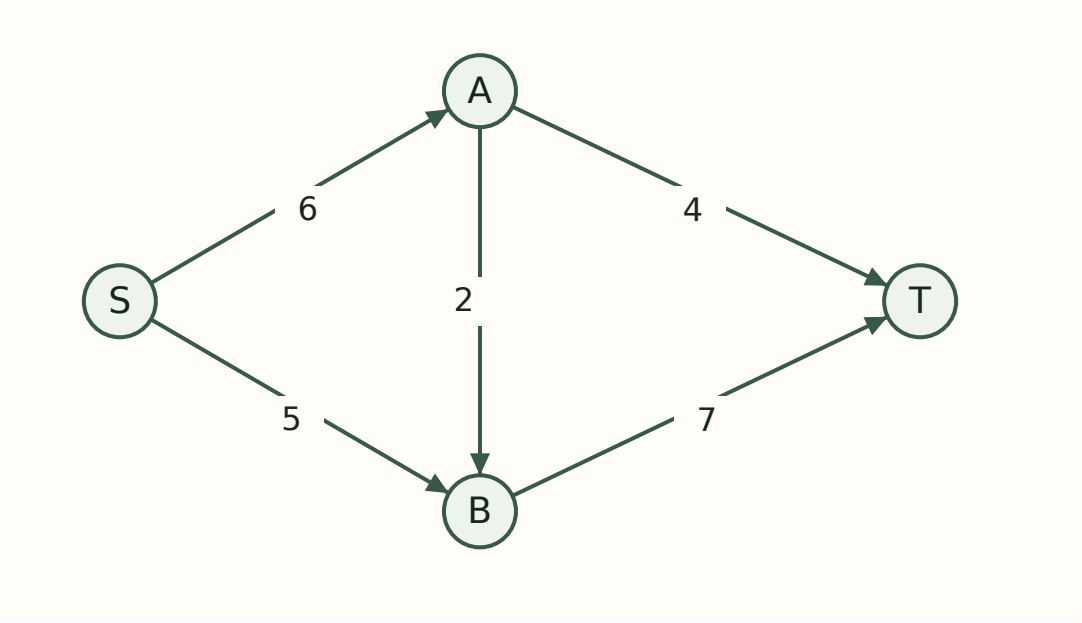
(b) State the installation cost and explain why adding B-D would stop the result being a tree. [2 marks]

$100(10) = 1000.$

B and D were already connected through C.

Answer: 1,000 dollars; adding B-D creates a cycle.

10. Capacities are litres per second. [4 marks]



(a) Give a feasible flow of 11 L/s, listing each edge flow. [2 marks]

At A, $6 = 2 + 4$.

At B, $5 + 2 = 7$.

The sink receives $4 + 7 = 11$.

Answer: SA 6, SB 5, AB 2, AT 4, BT 7.

(b) Use a cut to prove this is a maximum. [2 marks]

The only edges leaving S have capacities 6 and 5.

No flow can exceed their total 11.

Answer: The source cut has capacity 11 L/s, matching the feasible flow.