Practice test 5 - Applications and judgement

Name: _____ Date: _____

10 questions | 62 marks | 75 minutes

Combine algebra, changing tariffs, exponential fitting and model limitations in a more demanding paper.

A scientific calculator may be used.

1. Use inverse operations and a financial formula. [4 marks]

(a) Solve $4(2x-3)=5x+9$. [2 marks]

(b) In $I=Prt$, make r the subject. Find r if $I=180,\quad P=1500$ and $t=2$. [2 marks]

2. A line crosses the axes at $(6, 0)$ and $(0, -9)$. [6 marks]

(a) Find its equation. [2 marks]

(b) Find x when $y = 12$. [2 marks]

(c) Explain why its positive gradient is consistent with its negative y-intercept. [2 marks]

3. A hire plan A costs $20 + 4h$ dollars for $0 \le h \le 5$, then $40 + 2(h - 5)$ for $h > 5$. Plan B costs $10 + 5h$ for all $h \ge 0$. [9 marks]

(a) Check that plan A joins at $h = 5$. [2 marks]

(b) Find the equal-cost time, checking which interval contains the solution. [4 marks]

(c) Which plan is cheaper at 4 h and at 8 h? [3 marks]

4. A venue sells 30 tickets. Adult tickets cost 12 dollars, concession tickets 8 dollars, and revenue totals 304 dollars. [6 marks]

(a) Form simultaneous equations for adult count a and concession count c. [2 marks]

(b) Find the numbers sold. [2 marks]

(c) The venue cost is 180 dollars. Find the surplus and state one model assumption. [2 marks]

5. An exponential mass model has $P(2) = 450$ g and $P(5) = 777.6$ g. Its factor is positive. [6 marks]

(a) Find the per-step factor. [2 marks]

(b) Find a rule $P = Ab^t$. [2 marks]

(c) Predict the mass at $t = 7$. [2 marks]

6. A substance follows the continuous model $M = 80(0.95)^t$ grams, with t in weeks. [7 marks]

(a) Interpret the weekly factor. [2 marks]

(b) Find when the mass reaches half its initial value. [3 marks]

(c) At which first whole week is mass below 40 g? [2 marks]

7. Compare $L = 100 + 100t$ with $E = 100 \times 2^t$. [6 marks]

(a) Show they agree at $t = 0$ and $t = 1$. [2 marks]

(b) Compare them at $t = 0.5$ and $t = 2$. [2 marks]

(c) Correct the claim that exponential growth is larger at every positive time. [2 marks]

8. A process has observations $(0, 200), (1, 245), (2, 290), (3, 336)$. Fit using only the first three. [6 marks]

t	0	1	2	3
Observed y	200	245	290	336

(a) Find the linear rule for those first three points. [2 marks]

(b) Calculate the prediction and residual for the fourth point. [2 marks]

(c) Explain why this small residual alone does not establish a reliable forecast at $t = 100$. [2 marks]

9. Monthly sales are 1000,1100,1200,1300 units at months 0 to 3. A manager assumes 10% growth every month. [6 marks]

(a) Give a simple rule that fits all four observations exactly. [2 marks]

(b) Calculate the manager’s exponential prediction at month 3 and its error relative to the observation. [2 marks]

(c) Find the actual percentage increase from month 1 to 2. [2 marks]

10. A forecast is $P = 100(1.5)^n$ units for non-negative whole n. Available capacity is 700 units. [6 marks]

(a) Find P at $n = 4$ and $n = 5$. [2 marks]

(b) Find the last whole reporting input whose prediction is within capacity. [2 marks]

(c) State two useful planning responses rather than simply publishing the unrestricted forecast. [2 marks]