Practice test 5 - Applications and judgement

10 questions | 62 marks | 75 minutes

Combine algebra, changing tariffs, exponential fitting and model limitations in a more demanding paper.

Equivalent correct methods earn the corresponding marks.

1. Use inverse operations and a financial formula. [4 marks]

(a) Solve $4(2x-3)=5x+9$. [2 marks]

Expand: $8x-12=5x+9$.

Then $3x=21$.

Answer: $x=7$.

(b) In $I=Prt$, make r the subject. Find r if $I=180,\quad P=1500$ and $t=2$. [2 marks]

Divide by Pt.

$\frac{180}{(1500\times 2)}=0.06$.

Answer: $r=\frac{I}{(Pt)}=0.06$, or 6% per time unit.

2. A line crosses the axes at $(6,0)$ and $(0,-9)$. [6 marks]

(a) Find its equation. [2 marks]

Gradient $=\frac{(0-(-9))}{(6-0)}=1.5$.

The y-intercept is -9.

Answer: $y=1.5x-9$.

(b) Find x when $y=12$. [2 marks]

$12=1.5x-9$ gives $1.5x=21$.

Answer: $x=14$.

(c) Explain why its positive gradient is consistent with its negative y-intercept. [2 marks]

The line can start below zero and rise as x increases.

Answer: The gradient describes increase, while the intercept describes the value at zero.

3. A hire plan A costs $20+4h$ dollars for $0 \le h \le 5$, then $40+2(h-5)$ for $h>5$. Plan B costs $10+5h$ for all $h \ge 0$. [9 marks]

(a) Check that plan A joins at $h=5$. [2 marks]

First branch: $20+4(5)=40$.

Second expression: $40+2(0)=40$.

Answer: Both A branches give 40 dollars.

(b) Find the equal-cost time, checking which interval contains the solution. [4 marks]

First-branch equation gives $h=10$, outside 0-5, so reject it.

Later A branch is $30+2h$; equate to $10+5h$.

Then $20=3h$, giving $h=\frac{20}{3}>5$.

Answer: $h=\frac{20}{3}$ h, about 6.67 h.

(c) Which plan is cheaper at 4 h and at 8 h? [3 marks]

Use the correct branch for A at each input.

Compare with B at the same time.

Answer: At 4 h B is cheaper (30 versus 36 dollars); at 8 h A is cheaper (46 versus 50 dollars).

4. A venue sells 30 tickets. Adult tickets cost 12 dollars, concession tickets 8 dollars, and revenue totals 304 dollars. [6 marks]

(a) Form simultaneous equations for adult count a and concession count c. [2 marks]

The first equation counts tickets.

The second adds their prices.

Answer: $a+c=30$ and $12a+8c=304$.

(b) Find the numbers sold. [2 marks]

Substitute $c=30-a$.

$12a+8(30-a)=304$ gives $4a=64$.

Answer: 16 adult; 14 concession.

(c) The venue cost is 180 dollars. Find the surplus and state one model assumption. [2 marks]

$304-180=124$.

The revenue equations assume the listed prices apply.

Answer: 124 dollars; for example, every ticket was sold at the stated price with no refunds.

5. An exponential mass model has $P(2)=450$ g and $P(5)=777.6$ g. Its factor is positive. [6 marks]

(a) Find the per-step factor. [2 marks]

$b^3=\frac{777.6}{450}=1.728$.

Take the positive cube root.

Answer: 1.2.

(b) Find a rule $P=Ab^t$. [2 marks]

$A=\frac{450}{(1.2)^2}=312.5$.

Answer: $P=312.5(1.2)^t$.

(c) Predict the mass at $t=7$. [2 marks]

$312.5(1.2)^7=1119.744$.

Answer: 1119.744 g.

6. A substance follows the continuous model $M = 80(0.95)^t$ grams, with t in weeks. [7 marks]

(a) Interpret the weekly factor. [2 marks]

Subtract 0.95 from 1.

Answer: 95% remains; 5% is lost each week.

(b) Find when the mass reaches half its initial value. [3 marks]

$0.95^t = 0.5$.

$t = \frac{\log(0.5)}{\log(0.95)} \approx 13.5134$.

Answer: About 13.51 weeks.

(c) At which first whole week is mass below 40 g? [2 marks]

At 13, the fraction remaining is about 0.513342, above one-half.

At 14 it is about 0.487675, below one-half.

Answer: Week 14.

7. Compare $L = 100 + 100t$ with $E = 100 \times 2^t$. [6 marks]

(a) Show they agree at $t = 0$ and $t = 1$. [2 marks]

Substitute each input in each rule.

Answer: Both give 100 at 0 and 200 at 1.

(b) Compare them at $t = 0.5$ and $t = 2$. [2 marks]

$E(0.5) = 100\sqrt{2}$.

Evaluate both at 2.

Answer: At 0.5: $L = 150, \quad E \approx 141.42$. At 2: $L = 300, \quad E = 400$.

(c) Correct the claim that exponential growth is larger at every positive time. [2 marks]

One valid counterexample disproves the universal claim.

Answer: It is not true here, since at $t = 0.5$ the linear value is larger.

8. A process has observations $(0, 200), (1, 245), (2, 290), (3, 336)$. Fit using only the first three. [6 marks]

t	0	1	2	3
Observed y	200	245	290	336

(a) Find the linear rule for those first three points. [2 marks]

The consecutive difference is 45.

The initial value is 200.

Answer: $y = 200 + 45t$.

(b) Calculate the prediction and residual for the fourth point. [2 marks]

$200 + 45(3) = 335.$

$336 - 335 = 1.$

Answer: Prediction 335; residual 1.

(c) Explain why this small residual alone does not establish a reliable forecast at $t = 100$. [2 marks]

A close local fit does not establish that the rate remains constant for 100 steps.

Answer: The forecast is far outside the observed range and depends on untested assumptions.

9. Monthly sales are 1000,1100,1200,1300 units at months 0 to 3. A manager assumes 10% growth every month. [6 marks]

(a) Give a simple rule that fits all four observations exactly. [2 marks]

Consecutive differences are all 100.

Answer: $S = 1000 + 100n$.

(b) Calculate the manager's exponential prediction at month 3 and its error relative to the observation. [2 marks]

$1000(1.1)^3 = 1331.$

$1331 - 1300 = 31.$

Answer: 1331; it overpredicts by 31 units.

(c) Find the actual percentage increase from month 1 to 2. [2 marks]

Increase is 100 on a previous value 1100.

$\frac{100}{1100} \times 100\% \approx 9.09\%.$

Answer: About 9.09%.

10. A forecast is $P = 100(1.5)^n$ units for non-negative whole n. Available capacity is 700 units. [6 marks]

(a) Find P at $n = 4$ and $n = 5$. [2 marks]

Evaluate the rule at the two inputs.

Answer: 506.25 and 759.375.

(b) Find the last whole reporting input whose prediction is within capacity. [2 marks]

The positive-factor growth model increases.

The next reporting input already exceeds 700.

Answer: $n = 4$.

(c) State two useful planning responses rather than simply publishing the unrestricted forecast. [2 marks]

A feasible decision must respect 700 unless resources change.

Explain how changed capacity or restricted growth would alter the model.

Answer: For example, limit output at capacity and investigate the cost of expanding capacity.