Practice test 4 - Explain and correct

10 questions | 56 marks | 75 minutes

Find errors in algebra and modelling, support corrections and evaluate claims.

Equivalent correct methods earn the corresponding marks.

1. A student solves $5 - 2x = 17$ and reports $x = 6$. [4 marks]

(a) Find the correct solution. [2 marks]

Subtract 5: $-2x = 12$.

Divide by -2.

Answer: $x = -6$.

(b) Check both values and explain why the reported answer fails. [2 marks]

Substitute into the original equation.

The sign from division by -2 must be retained.

Answer: At $x = 6$ the left side is -7; at $x = -6$ it is 17.

2. A line passes through $(2, 12)$ and $(5, 24)$. A student writes $y = 4x + 12$. [4 marks]

(a) Check the gradient and find the correct rule. [2 marks]

$m = \frac{(24 - 12)}{(5 - 2)} = 4$.

Use $12 = 4(2) + b$ to get $b = 4$.

Answer: Gradient 4 is correct; rule is $y = 4x + 4$.

(b) Explain why 12 is not its y-intercept. [2 marks]

The y-intercept is the output when the input is zero.

Answer: 12 is the output at $x = 2$, not $x = 0$.

3. Consider the lines $y = 7$ and $x = 7$. [6 marks]

(a) State each gradient. [2 marks]

A horizontal line has zero vertical change.

A vertical line has zero horizontal change.

Answer: $y = 7$: gradient 0. $x = 7$: undefined.

(b) Find their intersection. [2 marks]

Both x and y must equal 7.

Answer: $(7, 7)$.

(c) Explain why neither is written as $y = mx + b$ with a finite nonzero m. [2 marks]

Vertical lines are not functions of x with one y per input.

Answer: The first has $m = 0$; the second cannot be expressed in that form with finite m.

4. Classify the solution sets of two systems. [6 marks]

(a) System A: $y = 3x - 4$ and $2y = 6x - 8$. [2 marks]

Divide the second rule by 2; the equations coincide.

Answer: Infinitely many common solutions.

(b) System B: $y = 3x - 4$ and $y = 3x + 1$. [2 marks]

The two lines have equal gradients and distinct intercepts.

Answer: No solution.

(c) Explain how the graph would confirm each conclusion. [2 marks]

Intersections represent shared solutions.

Answer: A: the same line; B: distinct parallel lines.

5. A service charges 12 dollars for up to 3 h, then 2 dollars per extra hour, proportionally. Someone uses $C = 12 + 2t$ for $t > 3$. [6 marks]

(a) Write the correct later rule. [2 marks]

The extra charge applies only to time beyond 3 h.

Answer: $C = 12 + 2(t - 3)$.

(b) Compare the correct and proposed costs at $t = 5$. [2 marks]

Correct: $12 + 2(2) = 16$.

Proposed: $12 + 2(5) = 22$.

Answer: Correct 16 dollars; proposed 22 dollars.

(c) Explain the discrepancy. [2 marks]

It adds 2×3 even though that time is already covered.

Answer: The proposed rule charges an extra 6 dollars for the first three hours.

6. A report says a multiplying factor of 1.2 means 120% annual growth and two years therefore means 40% total growth. [6 marks]

(a) Correct the annual growth claim. [2 marks]

1.2 retains the initial whole plus 0.2 extra.

Answer: The annual increase is 20%.

(b) Find the true two-year overall percentage increase. [2 marks]

$1.2^2 = 1.44$; the excess above 1 is 0.44.

Answer: 44%.

(c) An amount is 720 after those two years. Find its initial value. [2 marks]

$\frac{720}{1.44} = 500$.

Answer: 500.

7. A quantity falls from 1200 to 972 over two equal intervals. Assume fixed-percentage decay. [6 marks]

(a) Find its positive step factor and rate of decrease. [2 marks]

$b^2 = \frac{972}{1200} = 0.81$; use $b = 0.9$.

Answer: 0.9; 10% decrease.

(b) Find the first whole interval below 700. [2 marks]

At 5 it is $1200(0.9)^5 = 708.588$.

At 6 it is 637.7292.

Answer: Interval 6.

(c) Why is subtracting 120 each interval not the same model? [2 marks]

The second compound decrease is 108, not 120.

Answer: That removes a fixed amount rather than 10% of the current amount.

8. Values at $n = 0, \quad 1, \quad 2$ are 50,55,60.5. Someone says they cannot be exponential because they do not double. [6 marks]

(a) Find a constant-factor model. [2 marks]

Both consecutive ratios are 1.1.

Answer: $E = 50(1.1)^n$.

(b) Compare the $n = 2$ value with the linear rule $L = 50 + 5n$. [2 marks]

The two rules agree at $n = 0$ and 1 but not at 2.

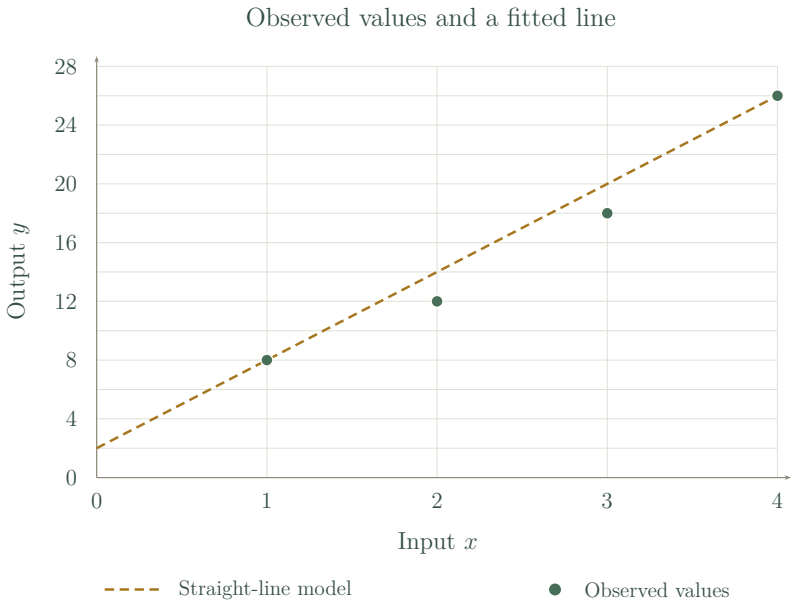
Answer: $E = 60.5; \quad L = 60$.

(c) Correct the claim about doubling. [2 marks]

Any fixed positive factor produces this form; here it is 1.1.

Answer: Exponential factors need not equal 2.

9. Use the graph below. Circles are observations; the dashed line is fitted through the first and last observations. Each small vertical grid step is 2 units. [6 marks]



(a) Read the first and last observations and find the fitted line's equation. [2 marks]

Read the endpoint coordinates from the graph.

Gradient $= \frac{(26-8)}{(4-1)} = 6$; use $8 = 6 + b$ to get $b = 2$.

Answer: $(1, 8)$ and $(4, 26)$; $y = 6x + 2$.

(b) Read all four observed outputs from the graph and find their residuals at $x = 1, \quad 2, \quad 3, \quad 4$. [2 marks]

The fitted outputs are 8,14,20,26.

Subtract each fitted output from its observed circle value.

Answer: Observed outputs 8,12,18,26; residuals 0, -2, -2, 0.

(c) Does joining the endpoints establish an exact linear model for these data? [2 marks]

The two middle residuals are -2, so the line does not fit every observation exactly.

Answer: No; the middle observations do not lie on the line.

10. A fitted electricity bill rule is $C = 30 + 0.25u$ dollars for observations between 100 and 600 kWh, all on the same tariff. [6 marks]

(a) Predict the bill at 400 kWh and classify it. [2 marks]

$30 + 0.25(400) = 130$.

400 is in the observed range.

Answer: 130 dollars; interpolation.

(b) Evaluate the rule at 5000 kWh. [2 marks]

$30 + 0.25(5000) = 1280$.

Answer: 1280 dollars.

(c) Explain why the second value needs caution. [2 marks]

The given data do not establish that the same rule applies at 5000 kWh.

Answer: It is distant extrapolation and may cross tariff or operating conditions.