Practice test 3 - Decisions from data

10 questions | 59 marks | 75 minutes

Use calibration, charges, production and investment models to make defensible decisions.

Equivalent correct methods earn the corresponding marks.

1. Solve and rearrange useful relationships. [4 marks]

(a) Solve $\frac{(x-5)}{3}=4$. [2 marks]

Multiply both sides by 3: $x-5=12$.

Add 5.

Answer: $x=17$.

(b) In $W=P(1+r)$, make r the subject. Then find r for $W=230$ and $P=200$. [2 marks]

Divide by P, then subtract 1.

$\frac{230}{200}-1=0.15$.

Answer: $r=\frac{W}{P}-1=0.15$, or 15%.

2. A sensor converts a raw reading r to a calibrated reading y. Its calibration gives $y=-2$ at $r=0$ and $y=18$ at $r=10$. [6 marks]

(a) Construct the straight-line calibration. [2 marks]

Gradient $=\frac{(18-(-2))}{10}=2$.

The intercept is -2.

Answer: $y=2r-2$.

(b) What raw reading corresponds to calibrated value zero? [2 marks]

Solve $0=2r-2$.

Answer: $r=1$.

(c) Find y when $r=7.5$. [2 marks]

$2(7.5)-2=13$.

Answer: 13.

3. A workshop sells each item for 24 dollars. Its cost is $C = 500 + 14n$ dollars and capacity is 60 items. [6 marks]

(a) Find break-even and the first profitable whole-number quantity. [2 marks]

$24n = 500 + 14n$ gives $n = 50$.

Profit $= 10n - 500$ is positive above 50.

Answer: 50 items; first profit at 51.

(b) How many items would give 250 dollars profit according to the rule? [2 marks]

$10n - 500 = 250$ gives $n = 75$.

Answer: 75 items.

(c) Can that profit be reached under the stated capacity? State the maximum modelled profit. [2 marks]

75 exceeds capacity.

$10(60) - 500 = 100$.

Answer: No; at 60 items profit is 100 dollars.

4. A group pays 12 dollars per person for its first five people and 8 dollars per additional person. [6 marks]

(a) Write the total-price rule for integer $n > 5$. [2 marks]

The first five cost 60 dollars.

Only later people receive the 8-dollar rate.

Answer: $C = 60 + 8(n - 5)$.

(b) Find the price for nine people. [2 marks]

$60 + 8(9 - 5) = 92$.

Answer: 92 dollars.

(c) A student proposes $C = 8n$ for every group. Explain the error and its effect for nine people. [2 marks]

The first five cost 12 each, not 8 each.

The missing amount is $5(12 - 8) = 20$.

Answer: It omits 20 dollars of earlier charges; it gives 72 rather than 92.

5. A vehicle starts at 30000 dollars and retains 85% of its value each year. [7 marks]

(a) Write a depreciation model. [2 marks]

A retention factor 0.85 is a 15% decrease.

Answer: $V = 30000(0.85)^n$.

(b) Find its value after four years, to cents. [2 marks]

$30000(0.85)^4 = 15660.1875$.

Answer: 15660.19 dollars.

(c) Find the first whole year in which it is worth less than 10000 dollars. [3 marks]

Year 6 is about 11314.49 dollars.

Year 7 is about 9617.31 dollars.

Answer: Year 7.

6. A data set has values 400 at $n = 0$ and 576 at $n = 2$. Both a linear and an exponential model are proposed. [6 marks]

(a) Fit a positive-factor exponential rule. [2 marks]

$b^2 = \frac{576}{400} = 1.44$, giving $b = 1.2$.

Answer: $E = 400(1.2)^n$.

(b) Fit a line through the same observations. [2 marks]

Gradient $= \frac{(576 - 400)}{2} = 88$.

Answer: $L = 400 + 88n$.

(c) Compare the models at $n = 4$. [2 marks]

Evaluate $400(1.2)^4$ and $400 + 88(4)$.

Answer: $E = 829.44$; $L = 752$; E is 77.44 higher.

7. Two accounts have models $A = 1000(1.05)^n$ and $B = 1100(1.03)^n$. Assume no fees, deposits or withdrawals. [6 marks]

(a) Compare balances after four years, to cents. [2 marks]

Evaluate both compound factors over four years.

Answer: $A = 1215.51$; $B = 1238.06$; B is larger.

(b) Compare balances after six years. [2 marks]

Evaluate both rules at $n = 6$.

Answer: $A = 1340.10$; $B = 1313.46$; A is larger.

(c) Explain why the larger starting balance does not settle every later comparison. [2 marks]

The smaller initial amount can overtake when its multiplying factor is larger.

Answer: The accounts have different percentage growth factors.

8. A linear relationship has observations $(1, 15), (3, 19), (5, 23)$. A later observation at $x = 7$ is 26. [6 marks]

(a) Fit the linear rule. [2 marks]

Gradient $= \frac{(19 - 15)}{2} = 2$.

Use $15 = 2 + 13$.

Answer: $y = 2x + 13$.

(b) Find the prediction and residual at $x = 7$. [2 marks]

$2(7) + 13 = 27$.

Residual $= 26 - 27 = -1$.

Answer: Prediction 27; residual -1.

(c) Does one nonzero residual justify deleting the observation? Explain. [2 marks]

It might be ordinary measurement variation or evidence about model fit.

Check data quality and context before excluding anything.

Answer: No.

9. A walker has distances d (km) at times t (h): $(0,0),(2,6),(5,6),(7,14)$. Assume straight segments between these observations. [6 marks]

Distance travelled

t (hours)	0	2	5	7
d (km)	0	6	6	14

(a) Find the speed over the first two hours and explain the segment from $t=2$ to 5. [2 marks]

First gradient $=\frac{6}{2}=3$.

Distance stays at 6 km from 2 to 5.

Answer: 3 km/h; then a three-hour stop.

(b) Write the distance rule for $5 \le t \le 7$. [2 marks]

The final segment gradient is $\frac{(14-6)}{(7-5)}=4$.

It starts at distance 6 when $t=5$.

Answer: $d=6+4(t-5)$.

(c) Find the average speed for the whole trip. [2 marks]

Total distance 14 km divided by total time 7 h.

Answer: 2 km/h.

10. Production is predicted by $N = 500(1.25)^n$ units/day after n weeks. The equipment can make at most 2000 units/day. [6 marks]

(a) Find predictions for weeks 6 and 7. [2 marks]

Evaluate the rule at each week without intermediate rounding.

Answer: About 1907.35 and 2384.19 units/day.

(b) Find the first whole week at which the prediction exceeds capacity. [2 marks]

Week 6 is below 2000; week 7 is above it.

Answer: Week 7.

(c) Give a practical consequence for using the model. [2 marks]

A revised model or additional capacity is needed; rounding alone does not solve the problem.

Answer: Unrestricted growth cannot continue on the same equipment beyond that bound.