Practice test 1 - Models and methods

10 questions | 58 marks | 60 minutes

A balanced first paper: solve, construct, compare and interpret linear, piecewise and exponential rules.

Equivalent correct methods earn the corresponding marks.

1. Work with equations and a delivery formula. [4 marks]

(a) Solve $4x - 7 = 25$. [2 marks]

Add 7 to obtain $4x = 32$.

Divide by 4.

Answer: $x = 8$.

(b) A delivery over distance d kilometres costs $C = 12 + 3d$ dollars. Find the distance for a 39-dollar bill. [2 marks]

$39 = 12 + 3d$ gives $3d = 27$.

Thus $d = 9$.

Answer: 9 km.

2. A straight line passes through $(0, 8)$ and $(4, 20)$. [6 marks]

(a) Find its gradient. [2 marks]

$m = \frac{(20 - 8)}{(4 - 0)} = 3$.

Answer: 3.

(b) Write its equation. [2 marks]

The y-intercept is 8.

Use $y = mx + b$.

Answer: $y = 3x + 8$.

(c) Find its x-intercept. [2 marks]

Set $y = 0$: $3x + 8 = 0$.

Thus $x = -\frac{8}{3}$.

Answer: $(-\frac{8}{3}, 0)$.

3. A bike-hire company charges 26 dollars for 2 h and 50 dollars for 6 h. Assume a fixed fee plus a constant hourly rate. [7 marks]

(a) Construct the cost rule for h hours. [3 marks]

Hourly rate $= \frac{(50-26)}{(6-2)} = 6$.

Use $26 = 12 + b$, so $b = 14$.

Answer: $C = 14 + 6h$.

(b) Find the cost of 3.5 h. [2 marks]

$14 + 6(3.5) = 35$.

Answer: 35 dollars.

(c) Find the hire time for a 62-dollar bill. [2 marks]

$62 = 14 + 6h$ gives $h = 8$.

Answer: 8 h.

4. A stall receives 15 dollars per item and has total cost $C = 120 + 7n$ dollars. [7 marks]

(a) Find the break-even quantity. [2 marks]

$15n = 120 + 7n$ gives $8n = 120$.

Thus $n = 15$.

Answer: 15 items.

(b) Find profit from 20 items. [2 marks]

Profit $= 15(20) - [120 + 7(20)] = 40$.

Answer: 40 dollars.

(c) Find the smallest whole number of items giving at least 100 dollars profit. [3 marks]

$8n - 120 \geq 100$ gives $n \geq 27.5$.

The smallest feasible integer is 28.

Answer: 28 items.

5. Water costs 4 dollars/kL for the first 5 kL and 2.50 dollars/kL above 5 kL. [6 marks]

(a) Write a cost rule for usage $u > 5$. [2 marks]

The first band costs $4(5) = 20$.

Only the excess is charged at 2.5 dollars/kL.

Answer: $C = 20 + 2.5(u - 5)$.

(b) Find the bill for 8 kL. [2 marks]

$20 + 2.5(8 - 5) = 27.5$.

Answer: 27.50 dollars.

(c) Explain whether the two cost branches join at 5 kL. [2 marks]

The first rule gives $4(5) = 20$.

The later rule gives $20 + 2.5(0) = 20$.

Answer: Yes; both give 20 dollars.

6. An amount follows $P = 250(1.04)^n$, where n is years. [4 marks]

(a) State the starting amount and annual percentage change. [2 marks]

At $n = 0$ the value is 250.

The excess factor above 1 is 0.04.

Answer: 250; 4% annual growth.

(b) Find the amount after 3 years, to two decimal places. [2 marks]

$250(1.04)^3 = 281.216$.

Round only the final answer.

Answer: 281.22.

7. A quantity follows $V = 180(0.75)^n$. [6 marks]

(a) State its percentage decrease per step. [2 marks]

The remaining proportion is 75%.

The removed proportion is 25%.

Answer: 25%.

(b) Find $V(2)$. [2 marks]

$180(0.75)^2 = 101.25$.

Answer: 101.25.

(c) Find the first whole step when V is less than 80. [2 marks]

At 2 the value is 101.25.

At 3 it is 75.9375, below 80.

Answer: Step 3.

8. Two sequences are recorded at $n = 0, \quad 1, \quad 2, \quad 3$. A: 80,90,100,110. B: 80,88,96.8,106.48. [6 marks]

(a) Choose a linear or exponential rule for each. [2 marks]

A has constant difference 10.

B has constant ratio 1.1.

Answer: $A = 80 + 10n$; $B = 80(1.1)^n$.

(b) Compare their predictions at $n = 4$. [2 marks]

Substitute $n = 4$ into both rules.

Answer: $A = 120; \quad B = 117.128; \quad A$ is larger.

(c) Explain why B can have constant percentage change but changing absolute increases. [2 marks]

Each increase is 10% of the previous term, which is itself increasing.

Answer: The percentage acts on a changing amount.

9. A data set is $(0, 30), (2, 42), (4, 54)$. A new observation at input 6 is 70. [6 marks]

(a) Fit a straight line to the first three points. [2 marks]

Gradient $= \frac{(42 - 30)}{2} = 6.$

The intercept is 30; the third point also fits.

Answer: $y = 30 + 6t.$

(b) Find the prediction and residual at input 6. [2 marks]

Prediction $= 30 + 6(6) = 66.$

Residual $= 70 - 66 = 4.$

Answer: Prediction 66; residual 4.

(c) Is the prediction at 6 interpolation relative to the first three points? [2 marks]

6 lies beyond the fitted input interval 0 to 4.

Answer: No; it is extrapolation.

10. An initially full tank is described by $V = 90 - 6t$ litres, with t in minutes. [6 marks]

(a) Find when it becomes empty and state the physical time domain. [2 marks]

Set $90 - 6t = 0$ to get $t = 15.$

Exclude negative time and negative volume.

Answer: 15 min; $0 \le t \le 15.$

(b) Evaluate the formula at 18 min and explain its practical meaning. [2 marks]

$90 - 6(18) = -18.$

The draining rule must stop when the tank is empty.

Answer: -18 L; not a feasible volume.

(c) State one assumption made before the tank empties. [2 marks]

The coefficient is treated as an unchanging rate.

Answer: For example, constant outflow of 6 L/min with no inflow.