Practical trigonometry - Practice test 5: applications and reasoning

10 questions | 58 marks | 75 minutes

Use degree mode. Give calculated lengths and angles to one decimal place unless stated otherwise; bearings use three digits to the nearest degree. In triangle ABC, a is opposite A, b opposite B and c opposite C. Show working, units and reasons for rejected solutions.

Equivalent correct methods earn the corresponding marks.

1. Similar signs have area ratio $\text{small} : \text{large} = 25 : 81$. The smaller sign has a side 7.5 cm and perimeter 22.5 cm. [5 marks]

(a) Find the length scale factor and matching large side. [3 marks]

The length factor is $\sqrt{\frac{81}{25}} = \frac{9}{5}$.

$\frac{7.5 \times 9}{5} = 13.5$.

Answer: $\frac{9}{5}$; 13.5 cm.

(b) Find the larger perimeter. [2 marks]

Perimeter scales linearly: $\frac{22.5 \times 9}{5} = 40.5$.

Answer: 40.5 cm

2. A right triangle has perpendicular legs 20 cm and 21 cm. [4 marks]

(a) Find the hypotenuse. [2 marks]

$c^2 = 400 + 441 = 841$.

$c = 29$.

Answer: 29 cm

(b) Find its two acute angles. [2 marks]

Use $\tan\theta = \frac{20}{21}$.

The second acute angle is the complement.

Answer: 43.6° and 46.4°.

3. A cable runs from a roof point 12 m above level ground to an anchor 20 m horizontally from the building. [6 marks]

(a) Find its length and its angle above horizontal. [3 marks]

$L = \sqrt{12^2 + 20^2}$.

$\theta = \tan^{-1}\left(\frac{12}{20}\right)$.

Answer: 23.3 m; 31.0°.

(b) The anchor moves 2 m farther away on the same straight ground line. How much longer must the cable become? [3 marks]

$\text{New length} = \sqrt{12^2 + 22^2}$.

Subtract the original unrounded length.

Answer: 1.7 m

4. An acute angle θ satisfies $\sin\theta = \cos 38^\circ$. [4 marks]

(a) Find θ and explain without trial and error. [2 marks]

For complementary acute angles, $\sin\theta = \cos(90^\circ - \theta)$.

$\theta = 90^\circ - 38^\circ = 52^\circ$.

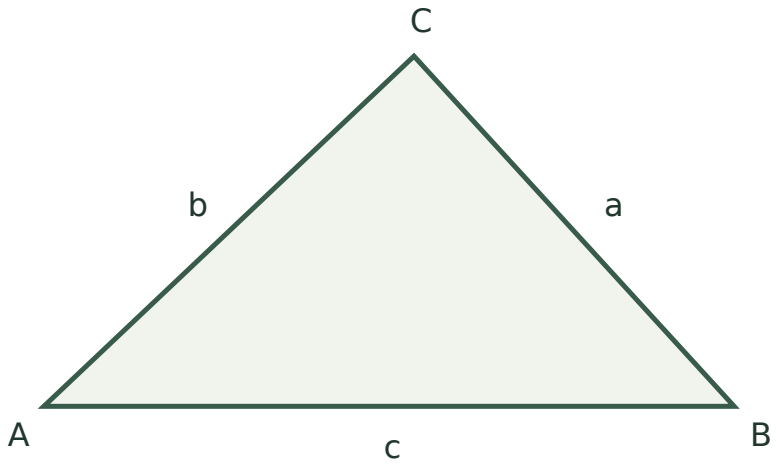
Answer: 52°

(b) Find $\tan\theta$ to three decimal places. [2 marks]

Evaluate $\tan 52^\circ$ in degree mode.

Answer: 1.280

5. Triangle ABC has $A = 40^\circ$, $a = 9\,\text{cm}$ and $b = 12\,\text{cm}$. [7 marks]



Sketch not to scale

(a) Find all possible pairs B,C. [4 marks]

$\sin B = \frac{12\sin 40^\circ}{9}$.

Use the acute candidate and its supplement; $C = 140^\circ - B$.

Answer: $B = 59.0^\circ$, $C = 81.0^\circ$; or $B = 121.0^\circ$, $C = 19.0^\circ$.

(b) Find c for each possible triangle. [3 marks]

For each C, use $c = \frac{9\sin C}{\sin 40^\circ}$.

Keep the two sets of angles and sides paired.

Answer: 13.8 cm and 4.6 cm.

6. A boat travels 10 km on bearing 060°, then 12 km on bearing 150°. [7 marks]

(a) Find its direct distance from the start. [3 marks]

The travel vectors are perpendicular because $150^\circ - 60^\circ = 90^\circ$.

$d = \sqrt{10^2 + 12^2}$.

Answer: 15.6 km

(b) Find the direct return bearing. [4 marks]

$E = 10\sin 60^\circ + 12\sin 150^\circ \approx 14.7;\ N = 10\cos 60^\circ + 12\cos 150^\circ \approx -5.4.$

The endpoint is south-east. Use the correct quadrant for the outward bearing, then reverse it.

Answer: 290°

7. A triangular panel has area $24\,\text{cm}^2$. Two sides are 8 cm and x cm, with included angle 30°. [4 marks]

(a) Find x. [2 marks]

$24 = \frac{1}{2} \times 8 \times x \times \sin 30^\circ = 2x.$

Thus $x = 12$.

Answer: 12 cm

(b) With those same two sides, find another included angle giving the same area. [2 marks]

The supplement has the same sine: $\sin 150^\circ = \sin 30^\circ$.

Answer: 150°

8. A triangular frame has sides 9 cm, 10 cm and 15 cm. [6 marks]

(a) Find its largest angle and state whether it is acute or obtuse. [3 marks]

The largest angle faces side 15.

$\cos C = \frac{81 + 100 - 225}{2 \times 9 \times 10} = -\frac{44}{180}$.

Answer: 104.1°; obtuse.

(b) Calculate its area. [3 marks]

The angle is included between sides 9 and 10.

$K = \frac{1}{2} \times 9 \times 10 \sin C$.

Answer: $43.6\,\text{cm}^2$

9. An instrument records $A = 65^\circ$, $a = 9\,\text{m}$ and $b = 14\,\text{m}$ for a proposed triangle. [6 marks]

(a) Demonstrate why the readings cannot all be correct. [3 marks]

$\sin B = \frac{14 \sin 65^\circ}{9} \approx 1.410$.

No angle has sine greater than 1.

Answer: $\sin B \approx 1.410 > 1$.

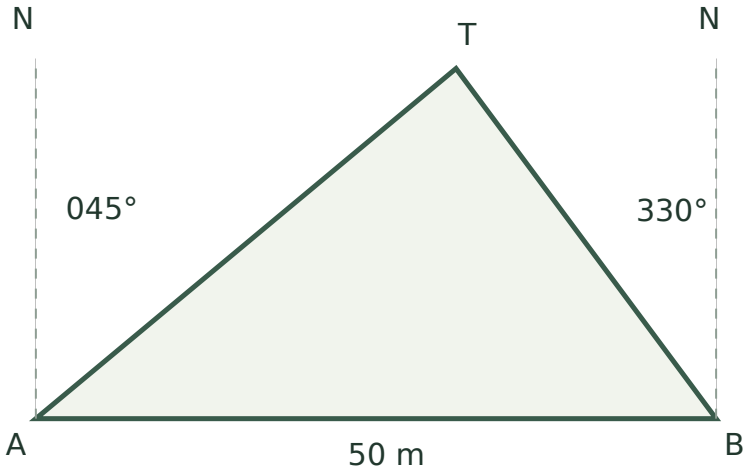
(b) If A and b are correct, find the smallest possible a. Describe the limiting triangle. [3 marks]

a must be at least $b \sin A = 14 \sin 65^\circ$.

At equality $B = 90^\circ$; hence $C = 25^\circ$.

Answer: 12.7 m; $B = 90^\circ$, $C = 25^\circ$.

10. On level ground, B is 50 m due east of A. A marker T is seen from A on bearing 045° and from B on bearing 330°. [9 marks]



Sketch not to scale

(a) Find all three interior angles of triangle ABT. [3 marks]

At A, angle between east 090° and T 045° is 45°.

At B, BA has bearing 270° and BT 330°, giving 60°.

$T = 180^\circ - 45^\circ - 60^\circ = 75^\circ$.

Answer: $A = 45^\circ$, $B = 60^\circ$, $T = 75^\circ$.

(b) Find AT and BT. [3 marks]

Use baseline 50 opposite angle 75°.

$AT = \frac{50 \sin 60^\circ}{\sin 75^\circ}$; $BT = \frac{50 \sin 45^\circ}{\sin 75^\circ}$.

Answer: $AT = 44.8\,\text{m}$; $BT = 36.6\,\text{m}$.

(c) Find perpendicular distance from T to AB, then the area of ABT. [3 marks]

$h = AT \sin 45^\circ$.

$K = \frac{1}{2} \times 50 \times h$, retaining precision until the end.

Answer: Height 31.7 m; area $792.5\,\text{m}^2$.