Practical trigonometry - Practice test 4: design and survey

10 questions | 50 marks | 75 minutes

Use degree mode. Give calculated lengths and angles to one decimal place unless stated otherwise; bearings use three digits to the nearest degree. In triangle ABC, a is opposite A, b opposite B and c opposite C. Show working, units and reasons for rejected solutions.

Equivalent correct methods earn the corresponding marks.

1. Two similar triangular panels have corresponding length ratio 3:7. The smaller panel has a side of 21 cm and area $45\,\text{cm}^2$. [4 marks]

(a) Find the corresponding larger side. [2 marks]

The enlargement factor is $\frac{7}{3}$.

$\frac{21\times 7}{3}=49$.

Answer: 49 cm

(b) Find the larger area. [2 marks]

$\text{Area factor}=(\frac{7}{3})^2=\frac{49}{9}$.

$\frac{45\times 49}{9}=245$.

Answer: $245\,\text{cm}^2$

2. A symmetrical roof section has horizontal span 10 m and vertical rise 3 m. A vertical line through the centre divides it into equal right triangles. [5 marks]

(a) Find each sloping roof length. [2 marks]

Each half has legs 5 m and 3 m.

$L=\sqrt{5^2+3^2}$.

Answer: 5.8 m

(b) Find the roof angle above horizontal and the triangular cross-sectional area. [3 marks]

$\theta=\tan^{-1}(\frac{3}{5})$.

$K=\frac{1}{2}\times 10\times 3=15\,\text{m}^2$.

Answer: $31.0°;\ 15\,\text{m}^2$.

3. A rectangular floor is 18 m by 24 m. A post 7 m high stands vertically at one corner. A cable runs from its top to the opposite floor corner. [5 marks]

(a) Find the floor diagonal. [2 marks]

$d = \sqrt{18^2 + 24^2} = 30.$

Answer: 30 m

(b) Find the cable length. [3 marks]

The vertical post and floor diagonal are perpendicular.

$L = \sqrt{30^2 + 7^2} = \sqrt{949}.$

Answer: 30.8 m

4. A ramp rises 0.75 m. Its design limit is a rise of 1 m for every 12 m horizontally. Use the limiting slope. [5 marks]

(a) Find the required horizontal run and the sloping length. [3 marks]

$\text{Horizontal run} = 0.75 \times 12 = 9\ \text{m}.$

$\text{Length} = \sqrt{9^2 + 0.75^2}.$

Answer: 9 m; 9.0 m.

(b) Find the ramp angle above horizontal. [2 marks]

$\tan\theta = \frac{0.75}{9} = \frac{1}{12}.$

Answer: 4.8°

5. An observer's eyes are 1.65 m above level ground. A building is 25 m horizontally away. Elevation to its roof is 22°. [4 marks]

(a) Find building height. [2 marks]

$\text{Rise above eyes} = 25\tan 22^\circ.$

$H = 1.65 + 25\tan 22^\circ.$

Answer: 11.8 m

(b) If the observer steps 5 m farther away along the same level line, does the elevation increase or decrease? Explain. [2 marks]

The same vertical rise is divided by a larger horizontal distance.

The tangent ratio, hence the acute elevation angle, decreases.

Answer: It decreases.

6. A vessel travels 12 km on bearing 350°, then 9 km on bearing 080°. [6 marks]

(a) Show the two travel directions are perpendicular, then find direct distance from the start. [3 marks]

The clockwise change across north is $10^\circ + 80^\circ = 90^\circ$.

$d = \sqrt{12^2 + 9^2} = 15\,\text{km}.$

Answer: 90° apart; direct distance 15 km.

(b) Find the final bearing from the start. [3 marks]

$E = 12\sin 350^\circ + 9\sin 80^\circ \approx 6.8\,\text{km}.$

$N = 12\cos 350^\circ + 9\cos 80^\circ \approx 13.4\,\text{km}.$

The point is north-east; $\text{bearing} = \tan^{-1}\left(\frac{E}{N}\right)$.

Answer: 027°

7. A triangular solar plot has sides 40 m and 55 m enclosing 70°. [5 marks]

(a) Calculate the area. [2 marks]

$K = \frac{1}{2} \times 40 \times 55 \sin 70^\circ$.

Answer: $1033.7\,\text{m}^2$

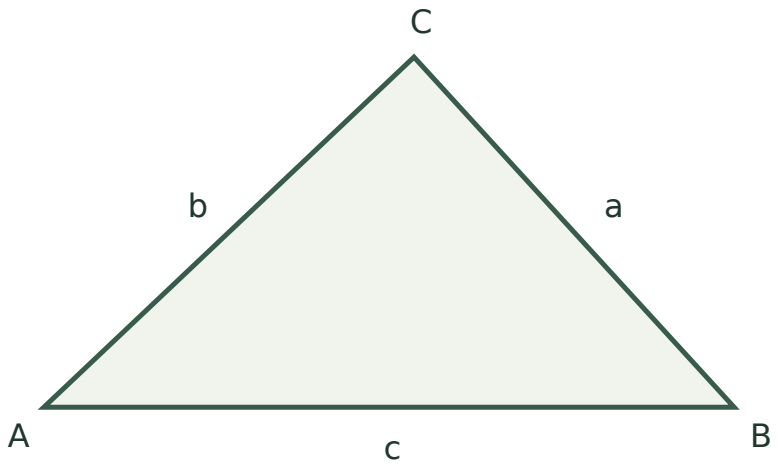
(b) Fencing is required on the third side only. Find that side and its cost at 18 dollars per metre. [3 marks]

Use cosine rule for the third side.

Multiply its unrounded length by 18 and round money to cents.

Answer: 55.9 m; 1005.44 dollars.

8. Triangle ABC has $A = 37^\circ$, $B = 83^\circ$ and $b = 18\,\text{m}$. [5 marks]



Sketch not to scale

(a) Find C and a. [3 marks]

$C = 180^\circ - 37^\circ - 83^\circ = 60^\circ$.

$a = \frac{18 \sin 37^\circ}{\sin 83^\circ}$.

Answer: 60°; $a = 10.9\,\text{m}$.

(b) Find c and check which side should be longest. [2 marks]

$c = \frac{18 \sin 60^\circ}{\sin 83^\circ}$.

$B = 83^\circ$ is the largest angle, so $b = 18\,\text{m}$ must be longest.

Answer: $c = 15.7\,\text{m}$; b is longest.

9. A triangle has sides 6 cm, 11 cm and 14 cm. [5 marks]

(a) Find its largest angle. [2 marks]

The largest angle is opposite side 14.

$\cos A = \frac{6^2 + 11^2 - 14^2}{2 \times 6 \times 11}$.

Answer: 107.2°

(b) Find its area without treating any two sides as perpendicular. [3 marks]

The angle found is included between sides 6 and 11.

$K = \frac{1}{2} \times 6 \times 11 \sin A$.

Answer: $31.5\,\text{cm}^2$

10. A 45 m straight baseline AB and marker P form a triangle, with $PAB = 48^\circ$ and $PBA = 72^\circ$. All measurements are horizontal. [6 marks]

(a) Find angle P and AP. [3 marks]

$P = 180^\circ - 48^\circ - 72^\circ = 60^\circ$.

$AP = \frac{45 \sin 72^\circ}{\sin 60^\circ}$.

Answer: 60°; $AP = 49.4\,\text{m}$.

(b) Find perpendicular distance from P to AB, and state one practical assumption. [3 marks]

$h = AP \sin 48^\circ$, using unrounded AP.

A suitable assumption connects the ideal triangle to the survey.

Answer: 36.7 m. Assume the baseline/sightings represent straight lines on a level horizontal plane.