Practical trigonometry - Practice test 3: connected problems

10 questions | 52 marks | 70 minutes

Use degree mode. Give calculated lengths and angles to one decimal place unless stated otherwise; bearings use three digits to the nearest degree. In triangle ABC, a is opposite A, b opposite B and c opposite C. Show working and units.

Equivalent correct methods earn the corresponding marks.

1. Similar displays have length ratio $\text{small} : \text{large} = 2 : 3$. The small display has a side 9 cm and area $28\,\text{cm}^2$. [4 marks]

(a) Find the matching large side. [2 marks]

$\text{Length factor} = \frac{3}{2}$.

$9 \times 1.5 = 13.5$.

Answer: 13.5 cm

(b) Find the large area. [2 marks]

$\text{Area factor} = (\frac{3}{2})^2 = \frac{9}{4}$.

$\frac{28 \times 9}{4} = 63$.

Answer: $63\,\text{cm}^2$

2. Triangle ABC is right-angled at C, with $AC = 15\,\text{cm}$ and $AB = 17\,\text{cm}$. [5 marks]

(a) Find BC and $\sin A$. [3 marks]

$BC = \sqrt{17^2 - 15^2} = 8.$

$\sin A = \frac{BC}{AB} = \frac{8}{17}.$

Answer: $BC = 8\,\text{cm};\ \sin A = \frac{8}{17}.$

(b) Find $\cos B$ and explain its relationship to $\sin A$. [2 marks]

A and B are complementary.

BC is adjacent to B but opposite A.

Answer: $\cos B = \frac{8}{17} = \sin A.$

3. A 13 m ladder slides on level ground against a vertical wall. Its foot moves from 5 m to 12 m from the wall. [4 marks]

(a) Find the original and final heights of its top. [2 marks]

$\text{Original} = \sqrt{169 - 25} = 12.$

$\text{Final} = \sqrt{169 - 144} = 5.$

Answer: 12 m and 5 m.

(b) Find the vertical drop. [2 marks]

$\text{Drop} = 12 - 5 = 7\,\text{m}$, not the final height alone.

Answer: 7 m

4. A right triangle has opposite leg 5 cm and adjacent leg 12 cm relative to θ. [4 marks]

(a) Find both acute angles. [2 marks]

$\theta = \tan^{-1}\left(\frac{5}{12}\right)$.

The other is $90^\circ - \theta$.

Answer: 22.6° and 67.4°.

(b) Explain the error in using $\sin^{-1}\left(\frac{5}{12}\right)$. [2 marks]

The hypotenuse is 13.

Sine would use $\frac{5}{13}$, whereas tangent uses $\frac{5}{12}$.

Answer: 12 is adjacent, not the hypotenuse.

5. Two ground-level observers and a tower base lie on one straight level line. Both observers are on the same side of the tower, 20 m apart. The nearer elevation is 60°, the farther 30°. [7 marks]

(a) Let x be the nearer distance. Form an equation. [3 marks]

Both sides equal the tower height.

Answer: $x\tan 60^\circ = (x+20)\tan 30^\circ$.

(b) Find the two distances and the tower height. [4 marks]

Solve the equation to obtain $x = 10$.

$h = 10\tan 60^\circ = 10\sqrt{3}$.

Answer: 10 m and 30 m; height 17.3 m.

6. A boat travels 10 km on bearing 030°, then 15 km due east. [6 marks]

(a) Find its final eastward and northward distances. [3 marks]

$\text{First east component} = 10\sin 30^\circ = 5.$

$\text{Final east} = 5 + 15 = 20;\ \text{north} = 10\cos 30^\circ.$

Answer: East 20 km; north 8.7 km.

(b) Find its direct return distance and bearing. [3 marks]

$\text{Distance} = \sqrt{20^2 + (10\cos 30^\circ)^2}.$

Find the outward bearing from north, then add 180°.

Answer: 21.8 km; 247°.

7. A 10 m diagonal divides a quadrilateral into two triangles. The perpendicular heights from the other vertices are 4 m and 7 m, on opposite sides of the diagonal. [4 marks]

(a) Find the two triangle areas. [2 marks]

$\frac{1}{2} \times 10 \times 4 = 20;\ \frac{1}{2} \times 10 \times 7 = 35.$

Answer: $20\,\text{m}^2$ and $35\,\text{m}^2$.

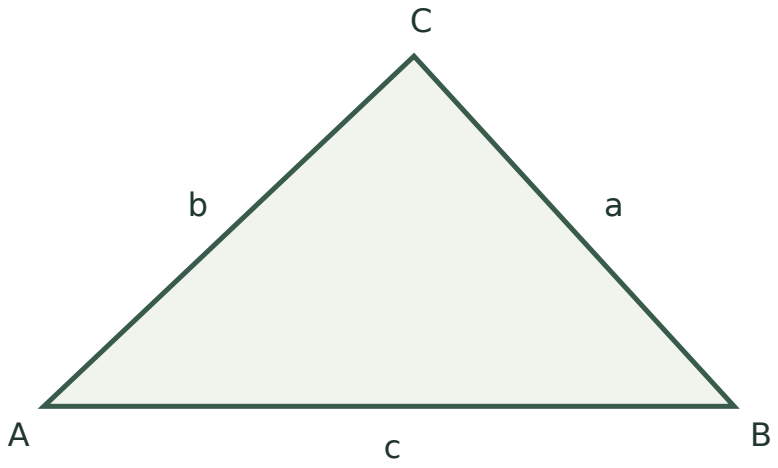
(b) Find total area and explain why an outside sloping side cannot replace either height. [2 marks]

Add 20+35.

A height must be perpendicular to the selected base.

Answer: $55\,\text{m}^2$

8. Triangle ABC has $A = 30^\circ$, $a = 10\,\text{cm}$ and $b = 14\,\text{cm}$. [7 marks]



Sketch not to scale

(a) Find every possible pair B,C. [4 marks]

$\sin B = 0.7$; take both sine candidates.

$C = 150^\circ - B$.

Answer: $B = 44.4^\circ$, $C = 105.6^\circ$; or $B = 135.6^\circ$, $C = 14.4^\circ$.

(b) Find c for each triangle. [3 marks]

$\frac{a}{\sin A} = 20$.

Use $c = 20\sin C$ for each C.

Answer: 19.3 cm and 5.0 cm.

9. Sides 8 cm and 13 cm enclose 45°. [5 marks]

(a) Find the third side. [2 marks]

$c^2 = 64 + 169 - 208\cos 45^\circ$.

Answer: 9.3 cm

(b) Find area and explain why these two sides are not Pythagoras legs. [3 marks]

$K = \frac{1}{2} \times 8 \times 13 \sin 45^\circ$.

The sides enclose 45°, not a right angle.

Answer: $36.8\,\text{cm}^2$

10. A survey triangle is proposed with $A = 45^\circ$, $a = 6\,\text{m}$ and $b = 10\,\text{m}$. [6 marks]

(a) Show that no triangle fits. [3 marks]

$\sin B = \frac{10 \sin 45^\circ}{6} \approx 1.1785$.

Answer: $\sin B \approx 1.179 > 1$; impossible.

(b) Find the smallest possible a for $A = 45^\circ$, $b = 10\,\text{m}$. Describe the boundary triangle. [3 marks]

Require $\frac{10 \sin 45^\circ}{a} \leq 1$.

At equality $\sin B = 1$, giving exactly one right triangle.

Answer: 7.1 m; $B = 90^\circ$.